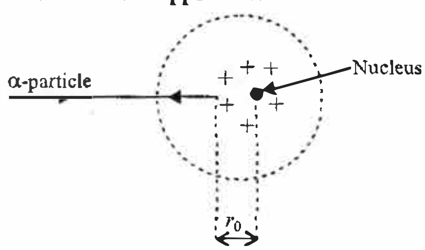
where K_α is the kinetic energy of the incident alpha particle.

- If $b = 0$, then by above relation
  $\cot\theta/2 = 0$ or $\theta/2 = 90°$ or $\theta = 180°$
  i.e., in case of head on collision, the impact parameter is zero and the alpha-particle rebounds back.
- If $b = \infty$, then by above relation
  $\cot\theta/2 = \infty$ or $\theta/2 = 0°$ or $\theta = 0°$
  i.e., the alpha particle goes nearly undeviated for a large impact parameter.

DISTANCE OF CLOSEST APPROACH : ESTIMATION OF NUCLEAR SIZE

- Suppose an α-particle of mass m and initial velocity v moves directly towards the centre of the nucleus of an atom. As it approaches the positive nucleus, it experiences Coulombic repulsion and its kinetic energy gets progressively converted into **electrostatic potential energy**. At a certain distance r_0 from the nucleus, the α-particle stops for a moment and then begin to retrace its path. The distance r_0 is called the **distance of closest approach.**



Distance of closest approach

- Let, initial kinetic energy of α-particle, $K_\alpha = \frac{1}{2}mv^2$
  Electrostatic potential energy of α-particle and nucleus at distance r_0,

$$U = \frac{q_1q_2}{4\pi\varepsilon_0 r_0} = \frac{2e.Ze}{r_0}\frac{1}{4\pi\varepsilon_0}$$

At the distance r_0, $K_\alpha = U$

$$\text{or} \quad \frac{1}{2}mv^2 = \frac{2e.Ze}{r_0}\frac{1}{4\pi\varepsilon_0}$$

$$r_0 = \frac{Ze^2}{\pi\varepsilon_0 mv^2}$$

Hence radius of nucleus must be smaller than r_0.

Illustration 1

In a Geiger-Marsden experiment, what is the distance of closest approach to the nucleus of a 7.7 MeV α-particle before it comes momentarily to rest and reverses its direction?

Soln.: The key idea here is that the total mechanical energy of the system consisting of an α-particle and a gold nucleus is conserved.

The initial energy E_i is just the kinetic energy K of the incoming α-particle. The final energy E_f is just the electric potential energy U of the system T.

Let d be the centre-to-centre distance between the α-particle and the gold nucleus when α-particle is at its stopping point. Then we can write the conservation of energy $E_i = E_f$ as

$$K_\alpha = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{r_0} = \frac{2Ze^2}{4\pi\varepsilon_0 r_0}$$

distance of closest approach

$$r_0 = \frac{2Ze^2}{4\pi\varepsilon_0 K}$$

here $K_\alpha = 7.7$ MeV $= 1.2 \times 10^{-12}$ J

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ MKS unit}$$

$Z = 79$ for Gold

$$\text{So,} \quad r_0 = \frac{2 \times 9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{1.2 \times 10^{-12}}$$

$r_0 = 3 \times 10^{-14}$ m $= 30$ fm.

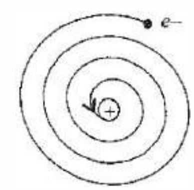
Radius of gold nucleus in actual is 6 fm where distance of closest approach is 30 fm. This discrepancy is due to the fact that distance of closest approach α is larger than sum of radii of the gold nucleus and the α particle.

Atomic Model from α-Particle Scattering Experiment

- The whole positive charge of atom is concentrated in its nucleus, which is of very small size as compared to size of atom. Atom has a diameter of the order of 10^{-10} m, whereas nucleus has a diameter of the order of 10^{-14} m.
- Electrons are situated in the large empty space around nucleus and are revolving, such that the centripetal force is provided by electrostatic force of attraction between electron and nucleus.
- The atom is neutral overall, so the total positive charge on nucleus is equal to the total negative charge on electrons.

Drawbacks of Rutherford s Model

- Rutherford's atomic model is inconsistent with classical physics. According to electromagnetic theory, an electron is a charged particle moving in the circular orbit around the nucleus and has an accelerated motion, so it should emit radiation continuously and thereby loose energy. Due to this, radius of the electron would decrease continuously and also the atom should then produce continuous spectrum, and ultimately electron will fall into the nucleus and atom will collapse in 10^{-8} s. But the atom is fairly stable and it emits line spectrum.



- Rutherford's model is not able to explain the spectrum of even most simplest H-spectrum.

Illustration 2

Answer the following questions, which help you to understand the difference between Thomson's model and Rutherford's model better.

(a) Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(b) Is the probability of backward scattering (*i.e.*, scattering of α-particles at angle greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(c) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

(d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Soln.: (a) Nearly the same. This is because we are considering the average angle of deflection.

(b) Much less, because there is no such massive core (nucleus) in Thomson's model as in Rutherford's model.

(c) This suggests that scattering is mainly due to a single collision, because the chance of a single collision increases linearly with the number of the target atoms, and hence linearly with the thickness of the foil.

(d) In Thomson model, positive charge is distributed uniformly in the atom. So single collision causes very little deflection. The observed average scattering angle can be explained only by considering multiple scattering. Hence it is wrong to ignore multiple scattering in Thomson's model.

BOHR MODEL OF HYDROGEN LIKE ATOMS

- Bohr postulated in his H-atom model.
  - The electron is revolving around the nucleus in stationary orbits.
  - When an electron makes a transition from a higher orbit to a lower stable orbit, the difference in the energy of the electron is radiated as a photon of energy $h\nu$.
  - The angular momentum of the electron in the stationary orbits is quantised.

    $$mvr = n\hbar \text{ where } \hbar = \frac{h}{2\pi}$$

    h is Planck's constant.

    n is called the quantum number;

    $n = 1$ for the first stable orbit, 2 for the second orbit, etc.
- The Bohr model is applicable not only for hydrogen but all hydrogen-like atoms *i.e.*, atoms which have been ionized to have a single electron revolving round the nucleus.

Circular Orbits

- The atom consists of central nucleus, containing the entire positive charge and almost all the mass of the atom. The electrons revolve around the nucleus in certain discrete circular orbits. The necessary centripetal force for circular orbits is provided by the Coulomb attraction between the electron and nucleus. So,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2}$$

where, m = mass of electron

r = radius of circular orbit,

v = speed of electron in circular orbit,

Ze = charge on nucleus,

Z = atomic number,

e = charge on electron = -1.6×10^{-19} C

Stationary Orbits

- The allowed orbits for the electrons are those in which the electron does not radiate energy. These orbits are also called stationary orbits.

Quantum Condition (Bohr's Quantisation Rule)

- The stationary orbits are those in which angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}(=\hbar)$

  i.e., $mvr = n\left(\frac{h}{2\pi}\right)$, n being integer or the principle quantum number.

Radius of Orbit

Since, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \quad \text{...(i)}$$

and $$mvr = \frac{nh}{2\pi} \quad \text{...(ii)}$$

Putting in (i), we get

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2Z} \Rightarrow r_n = (0.53)\frac{n^2}{Z}\text{ Å}$$

From (ii), $v = \dfrac{nh}{2\pi mr}$

So, for H-like atoms, $r_n \propto \dfrac{n^2}{Z}$.

Velocity of Electron in n^{th} Orbit

Since $v = \dfrac{nh}{2\pi mr}$ and $r = \dfrac{n^2h^2\varepsilon_0}{\pi me^2Z}$

$$\Rightarrow v = \left(\frac{e^2}{2h\varepsilon_0}\right)\frac{Z}{n} \Rightarrow v = \left(\frac{e^2}{2h\varepsilon_0 c}\right)\left(\frac{cZ}{n}\right)$$

where $\alpha = \dfrac{e^2}{2h\varepsilon_0 c}$ is the Sommerfeld's fine structure constant (a pure number) whose value is $\dfrac{1}{137}$.

$$\Rightarrow v = \left(\frac{1}{137}\right)\frac{cZ}{n}$$