velocities vary from 0.01-0.1 times of c (velocity of light). They can be deflected by electric and magnetic fields and have low penetrating power but high ionizing power.

- β-Particles ($_{-1}e^0$)
  - These are fast moving electrons having charge equal to $-e$ and mass $m_e = 9.1 \times 10^{-31}$ kg. Their velocities vary from 1% to 99% of the velocity of light (c). They can also be deflected by electric and magnetic fields. They have low ionizing power but high penetrating power. β^+ particles are positrons.
- γ-Radiation ($_0\gamma^0$)
  - These are electromagnetic waves of nuclear origin and of very short wavelength. They have no charge and no mass. They have maximum penetrating power and minimum ionising power. The energy released in a nuclear reaction is mainly emitted in the form of γ radiation.

LAWS OF RADIOACTIVE DECAY

Rutherford-Soddy Laws (Statistical Laws)

- The disintegration of a radioactive substance is random and spontaneous.
- Radioactive decay is purely a nuclear phenomenon and is independent of any physical and chemical conditions.

  The radioactive decay follows first order kinetics, *i.e.*, the rate of decay is proportional to the number of undecayed atoms in a radioactive substance at any time t. If dN be the number of atoms (nuclei) disintegrating in time dt, the rate of decay is given as dN/dt. From first order of kinetic rate law $\frac{dN}{dt} = -\lambda N$, where λ is called as decay or disintegration constant.
- Let N_0 be the number of nuclei at time $t = 0$ and N_t be the number of nuclei after time t, then according to integrated first order rate law, we have

$$N_t = N_0 e^{-\lambda t} \Rightarrow \lambda t = \ln\frac{N_0}{N_t} = 2.303\log\frac{N_0}{N_t}$$

- The half life ($T_{1/2}$) period of a radioactive substance is defined as the time in which one-half of the radioactive substance is disintegrated. If N_0 be the number of radioactive nuclei at $t = 0$, then in a half life $T_{1/2}$, the number of nuclei decayed will be $N_0/2$.

$$N_t = N_0 e^{-\lambda t} \quad ...(i)$$

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{N_t}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} = \left(\frac{1}{2}\right)^n$$

n = number of half lives

- The mean life (T_m) of a radioactive substance is equal to the sum of life times of all atoms divided by the number of all atoms. It is given by

$$T_m = \frac{1}{\lambda}$$

Illustration 7

The mean lives of a radio active substance are 1620 and 405 years for α-emission and β-emission respectively. Find out the time during which three fourth of a sample will decay if it is decaying both the α-emission and β-emission simultaneously.

Soln.: When a substance decays by α and β emission simultaneously, the average rate of disintegration λ_{av} is given by

$\lambda_{av} = \lambda_\alpha + \lambda_\beta$

where λ_α = disintegration constant for α-emission only.

λ_β = disintegration constant for β-emission only.

Mean life is given by

$$T_m = \frac{1}{\lambda}$$

$$\Rightarrow \lambda_{av} = \lambda_\alpha + \lambda_\beta \Rightarrow \frac{1}{T_m} = \frac{1}{T_\alpha} + \frac{1}{T_\beta}$$

$$= \frac{1}{1620} + \frac{1}{405} = 3.08 \times 10^{-3}$$

$$\lambda_{av} t = 2.303 \log\frac{100}{25}$$

$$(3.08 \times 10^{-3})t = 2.303 \log\frac{100}{25}$$

$$\Rightarrow t = 2.303 \times \frac{1}{3.08 \times 10^{-3}} \log 4 = 450.17 \text{ years.}$$

Soddy Fajan Laws (Group-Displacement Laws)

- When a nuclide emits one α-particle ($_2He^4$), its mass number (A) decreases by 4 units and atomic number (Z) decreases by 2 units.

$$_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 + \text{Energy}$$

- When a nuclide emits a β-particle, its mass number remains unchanged but atomic number increases by one unit.

$$_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\upsilon} + \text{Energy}$$

  where $\bar{\upsilon}$ antineutrino.
- In the nucleus, due to conversion of neutron into proton, antineutrino is produced. It has no charge or mass, but has momentum. When a proton is converted to a neutron, a neutron and a +ve β-particle is produced, which is called as positron. β rays are electrons and β^+ are the antielectrons or positrons.

$$_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 + \bar{\upsilon}\,(\text{antineutrino})$$

$$_1p^1 \rightarrow {}_0n^1 + {}_{+1}e^0\,(\text{positron}) + \upsilon(\text{neutrino})$$

- Antineutrino and neutrino share the energy of electrons and positrons. That is the reason why the energy of β is continuous and β rays has a maximum energy.

  When a γ particle is produced, both atomic and mass number remain constant.

Activity of a Radioactive Isotope

- The activity of a radioactive substance (or radioisotope) means the rate of decay per second or the number of nuclei disintegrating per second. It is generally denoted by A.

$$A = -\frac{dN}{dt}$$

- If at time $t = 0$, the activity of a radioactive substance be A_0 and after time $t = t$ s, activity be A_t then

$$A_0 = \left[\frac{dN}{dt}\right]_{t=0} = -\lambda N_0$$

$$A_t = \left[\frac{dN}{dt}\right]_{t=t} = -\lambda N_t$$

$$A_t = A_0 e^{-\lambda t}$$

Unit of Activity

- The activity is measured in terms of Curie (Ci). 1 curie is the activity of 1 g of a freshly prepared sample of radium Ra^{226} ($T_{1/2} = 1602$ years.)
  1 curie = 1 Ci = 3.7×10^{10} dps (disintegration per second)
  1 dps is also known as 1 Bq (becquerel).
  1 Ci = 3.7×10^{10} Bq

Illustration 8

Radioisotopes of phosphorus P^{32} and P^{35} are mixed in the ratio of 2:1 of atoms. The activity of the sample is 2 Ci. Find the activity of the sample after 30 days. $T_{1/2}$ of P^{32} is 14 days and $T_{1/2}$ of P^{35} is 25 days.

Soln.: Let A_0 = initial activity of sample.

A_{10} = initial activity of isotope 1 and

A_{20} = initial activity of isotope 2.

$A_0 = A_{10} = A_{20}$

Similarly for final activity (Activity after time t)

$A_t = A_{1t} = A_{2t}$

$\Rightarrow A_t = A_{10}e^{-\lambda_1 t} + A_{20}e^{-\lambda_2 t}$

Now in the given equation,

$$A_0 = 2\,\text{Ci} \Rightarrow A_0 = A_{10} + A_{20} = 2 \quad \text{...(i)}$$

Initial ratio of atoms of isotopes = 2 : 1

From definition of activity,

$A = \lambda N$

$$\Rightarrow \frac{A_{10}}{A_{20}} = \frac{\lambda_1 N_{10}}{\lambda_2 N_{20}} = \frac{N_{10}}{N_{20}} \times \frac{T_2}{T_1}$$

where T represents half life

$$\Rightarrow \frac{A_{10}}{A_{20}} = \frac{2}{1} \times \frac{25}{14} = \frac{50}{14} = \frac{25}{7} \quad \text{...(ii)}$$

On solving equation (i) and (ii), we get

$$A_{20} = \frac{7}{16} \text{ and } A_{10} = \frac{25}{16}$$

$$A_t = A_{10}e^{-\lambda_1 t} + A_{20}e^{-\lambda_2 t}$$

$$\Rightarrow A_t = \frac{25}{16}e^{-\frac{0.693}{14}\times 30} + \frac{7}{16}e^{-\frac{0.693}{25}\times 30}$$

Consider the first exponential term:

$$e^{-\frac{0.693\times 30}{14}} = e^{-1.485}$$

Let $y = e^{-1.485} \Rightarrow \ln y = -1.485$

$$\Rightarrow \log y = \frac{-1.485}{2.303} \Rightarrow y = \text{antilog}\left(\frac{-1.485}{2.303}\right)$$

So, from above calculations you can derive a general result

$$i.e.,\ e^{-x} = \text{antilog}\left(\frac{-x}{2.303}\right)$$

$$A_t = \frac{25}{16} \times 0.2265 + \frac{7}{16} \times 0.4354 = 0.5444 \text{ Ci.}$$

Illustration 9

A count- rate meter is used to measure the activity of a given sample. At one instant the meter shows 4750 counts per minute. Five minutes later it shows 2700 counts per minute. Find

(a) decay constant

(b) the half life of the sample.

Soln.: (a) Initial activity = $A_0 = -\frac{dN}{dt}$ at $t = 0$

Final activity = $A_t = -\frac{dN}{dt}$ at $t = t$

$$\left.\frac{dN}{dt}\right|_{t=0} = -\lambda N_0 \text{ and } \left.\frac{dN}{dt}\right|_{t=5} = -\lambda N_t \Rightarrow \frac{4750}{2700} = \frac{N_0}{N_t}$$

Using $\lambda t = 2.303 \log \frac{N_0}{N_t}$

$$\Rightarrow \lambda(5) = 2.303 \log\left(\frac{4750}{2700}\right)$$

$$\Rightarrow \lambda = \frac{2.303}{5}\log\left(\frac{4750}{2700}\right) = 0.1129 \text{ min}^{-1}$$

(b) $T_{1/2} = \frac{0.693}{0.1129} = 6.14$ min.

MASS DEFECT AND BINDING ENERGY

- The nucleons are bound together in a nucleus and the energy has to be supplied in order to break apart the constituents into free nucleons. The energy with which nucleons are bound together in a nucleus is called **binding energy** (B.E.). In order to free nucleons from a bound nucleus, this much of energy (= B.E.) has to be supplied.
- It is observed that the mass of a nucleus is always less than the mass of its constituent (free) nucleons. This difference in mass is called as **mass defect** and is denoted as Δm.
  If m_n = mass of neutron and m_p = mass of proton
  $M(Z, A)$ = mass of bound nucleus
  Then, $\Delta m = Zm_p + (A - Z)m_n - M(Z,A)$
- This mass defect is in form of energy and is responsible for binding the nucleons together. From Einstein's mass-energy relation,
  $E = \Delta mc^2$ (c speed of light, m is mass)
  $\Rightarrow$ Binding energy = Δmc^2
- Generally, Δm is measured in amu (u) units. So let us calculate the energy equivalent to 1 u. It is calculated in eV (electron volt, 1 eV = 1.6×10^{-19} J)

$$E(=1\text{ u}) = \frac{1 \times 1.67 \times 10^{-27} \times (3 \times 10^8)^2}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 931 \times 10^6 \text{ eV} = 931 \text{ MeV}$$