Illustration 2

The acceleration due to gravity is 9.8 m s⁻². Give its value in ft s $^{-2}$.

Soln. : As 1 m = 3.2 ft

 \therefore 9.8 m s⁻² = 9.8 × 3.28 ft/sec² = 32.14 ft s⁻² ≈ 32 ft s⁻².

DIMENSION OF A PHYSICAL QUANTITY

• All physical quantities can be expressed in terms of the seven fundamental units. We call these seven physical quantities as seven dimensions of the physical world. So, dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represented that quantity. For example,

Acceleration = $\frac{\text{Velocity}}{\text{Time}}$ \therefore Dimensions of acceleration = $\frac{\text{Velocity dimensions}}{\text{Time dimensions}}$

Dimension of Velocity = $\frac{\text{Dimension of length}}{\text{Dimension of time}} = \frac{L}{T} = [LT^{-1}]$

: Dimensions of acceleration

$$=\frac{LT^{-1}}{T}=LT^{-2}=M^{0}L^{1}T^{-2}$$

Hence the dimensions of acceleration are zero in mass, 1 inch length and -2 in time.

Dimensional Formula and Dimensional Equation

• The dimensional formula of any physical quantity is that expression which represents how and which of the base quantities are included in that quantity. It is written by enclosing the symbols for base quantities with appropriate powers in square brackets i.e. [⁻]

e.g. Dimensional formula of acceleration is $[M^{\bullet}L^{1}T^{-2}]$.

- The equation obtained by equating a physical quantity with its dimensional formula is called a **dimensional equation**.
 - e.g. Dimensional equation for acceleration is

$$[a] = [M^{\bullet}L^{1}T^{-2}]$$

Similarly, dimensional equation for density is $[\rho] = [M^{1}L^{-3}T^{0}]$

The dimensional formulae and SI units of physical quantities are as shown in the table.

S. No.	Physical quantity	Relation with other physical quantities	Dimensions	Dimensional formula	SI unit
1.	Area	Length × Breadth	[L] × [L]	$[M^{0}L^{2}T^{0}]$	m ²
2.	Volume	Length × Breadth × Height	$[L] \times [L] \times [L]$	$[M^0L^3T^0]$	m ³
3.	Density	Mass Volume	$\frac{[M]}{[L^3]}$	[ML- ³ T ⁰]	kg m ^{_3}
4.	Frequency	1 Time period	1 [T]	[M [●] L ⁰ T ⁻¹]	s ⁻¹ or Hz (hertz)
5.	Speed/Velocity	Distance/Displacement Time	[L] [T]	[M ⁰ LT ⁻¹]	m s ⁻¹
6.	Acceleration	Velocity Time	$\frac{[LT^{-1}]}{[T]}$	[M ⁰ LT ⁻²]	m s ⁻²
7.	Force	Mass × Acceleration	[M][LT ⁻²]	[MLT ⁻²]	N (newton)
8.	Impulse	Force × Time	[MLT-2][T]	[ML 1 ⁻¹]	N s
9.	Work	Force × Distance	[MLT ⁻²][L]	[ML ² T- ²]	J (joule)
10.	Energy	Work	[ML ² T ⁻²]	[ML ² T ⁻²]	J
11.	Power	Work Time	$\frac{[ML^2T^{-2}]}{[T]}$	[ML ² T-3]	W (watt)
12.	Momentum	Mass × Velocity	[M][LT ⁻¹]	[MLT-1]	kg m s ⁻¹
13.	Pressure, stress	Force Area	$\frac{[MLT^{-2}]}{[L^2]}$	[ML ⁻¹ T ⁻²]	N m ⁻² or Pa (Pascal)
14.	Strain	Change in dimension Original dimension	[L] [L]	[M⁰LºT⁰]	No units
15.	Modulus of elasticity	<u>Stress</u> Strain	$\frac{[\mathbf{M}\mathbf{L}^{-1}\mathbf{T}^{-2}]}{[\mathbf{M}^{0}\mathbf{L}^{0}\mathbf{T}^{0}]}$	$[\mathbf{M}\mathbf{L}^{-1}\mathbf{T}^{-2}]$	N m ⁻²
16.	Surface tension	Force Length	$\frac{[MLT^{-2}]}{[L]}$	[ML ⁰ T ⁻²]	N m ⁻¹

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	17.	Specific gravity/relative density	Density of body Density of water at 4°C	[ML ⁻³] [ML ⁻³]	[MºL.ºTº]	No units
	18.	Velocity gradient	Velocity Distance	[<u>LT⁻¹]</u> [L]	[[M ⁰ L ⁰ T ⁻¹]	s_1
	19.	Pressure gradient	Pressure Distance	$\frac{[ML^{-1}T^{-2}]}{[L]}$	[ML ⁻² T ⁻²]	N m -3
	20.	Pressure energy	Pressure × Volume	$[ML^{-1}T^{-2}][L^3]$	[ML ² T ⁻²]	J
	21.	Coefficient of viscosity	Force Area × velocity gradient	$\frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]}$	[ML-1T-1]	Pas (Pascal second)
	22.	Angle, Angular displacement	Arc Radius	[L]	[M ⁰ L ⁰ T ⁰]	rad (radian)
	23.	Trigonometric ratio (sinθ, cosθ, tanθ etc.)	Length Length	[L] [L]	[MºLºTº]	No units
	24.	Angular velocity	Angle Time	$\frac{[M^0L^0T^0]}{[T]}$	$[M^0L^0T^{-1}]$	rad s ⁻¹
	25.	Angular acceleration	Angular velocity Time	$\frac{[T^{-1}]}{[T]}$	[M ⁰ L ⁰ T ⁻²]	rad s-2
	26.	Radius of gyration	Distance	[L]	[M ⁰ LT ⁰]	m
	27.	Moment of inertia	Mass × (Radius of gyration) ²	[M][L ²]	$\left [ML^2T^0] \right $	kg m ²
	28.	Angular momentum	Moment of inertia × Angular velocity	[ML ²][T ⁻¹]	[ML ² T J]	kg m ² s -1
	29.	Moment of force, moment of couple	Force × Distance	[MLT-2][L]	[ML ² T- ²]	N m
	30.	Torque	Force × Distance	[MLT ⁻²][L]	[ML ² T- ²]	N m
-	31.	Angular frequency	$2\pi \times Frequency$	[T ⁻¹]	$[M^0L^0T^{-1}]$	rad s ⁻¹
	32.	Wavelength	Distance	[L]	[M ⁰ LT ⁰]	m
	33.	Hubble constant	Recession speed Distance	<u>[LT⁻¹]</u> [L]	[M ⁰ L ⁰ T ⁻¹]	s ⁻¹
	34.	Intensity of wave	Energy Time × area	$\frac{[ML^2T^{-2}]}{[T][L^2]}$	[ML ⁰ F ³]	W m ⁻²
	35.	Radiation pressure	Intensity of wave Speed of light	$\frac{[ML^{0}T^{-3}]}{[LT^{-1}]}$	[ML- ¹ T- ²]	N m- ²
	36.	Energy density	Energy Volume	$\frac{[ML^2T^{-2}]}{[L^3]}$	[ML- ¹ T- ²]	J m_3
	37.	Critical velocity	Reynold's number × Coefficient of viscosity Density × Radius	$\frac{[M^{0}L^{0}T^{0}][ML^{1}T^{-1}]}{[ML^{-3}][L]}$	[M ⁰ LT ⁻¹]	m s_1

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38.	Escape velocity	$(2 \times Acceleration due to gravity \times Earth's radius)^{1/2}$	$[LT^{-2}]^{1/2} \times [L]^{1/2}$	[M ⁰ LT ⁻¹]	m s ⁻¹
39.	Heat energy, internal energy	Energy	[ML ² T ⁻²]	[ML ² T ⁻²]	J
40.	Kinetic energy	$\frac{1}{2} \times \text{Mass} \times (\text{Velocity})^2$	[M][LT-1] ²	[ML ² T ⁻²]	J
41.	Potential energy	Mass × Acceleration due to gravity × Height	[M][LT ⁻²][L]	[ML ² T ⁻²]	J
42.	Rotational kinetic energy	$\frac{1}{2} \times \text{Moment of inertia} \times (\text{Angular velocity})^2$	[ML ²][T ⁻¹] ²	[ML ² T ⁻²]	J
43.	Efficiency	Output work or energy Input work or energy	$\frac{[ML^2T^{-2}]}{[ML^2T^{-2}]}$	[M ⁰ L ⁰ T ⁰]	No units
44.	Angular impulse	Torque × Time	[ML ² T ⁻²][T]	[ML ² T ⁻¹]	kg m ² s ⁻¹
45.	Gravitational constant	$\frac{\text{Force} \times (\text{Distance})^2}{\text{Mass} \times \text{Mass}}$	$\frac{[MLT^{-2}][L^{2}]}{[M][M]}$	[M ⁻¹ L ³ T ⁻²]	N m ² kg ⁻²
46.	Planck's constant	Energy Frequency	$\frac{[ML^2T^{-2}]}{[T^{-1}]}$	[ML ² T ⁻¹]	Js
47.	Heat capacity, entropy	Heat energy Temperature	$\frac{[ML^2T^{-2}]}{[K]}$	[ML ² T ⁻² K ⁻¹]	J K-1
48.	Specific heat capacity	Heat energy Mass × Temperature	$\frac{[ML^2T^{-2}]}{[M][K]}$	[M [•] L ² T ⁻² K ⁻¹]	J kg ⁻¹ K ⁻¹
49.	Latent heat	Heat energy Mass	$\frac{[ML^2T^{-2}]}{[M]}$	[M [•] L ² T ⁻²]	J kg ⁻¹
50.	Thermal expansion coefficient or thermal expansivity	Change in dimension Original dimension × Temperature	[L] [L][K]	[M●L ⁰ T ⁰ K ⁻¹]	K-1
51.	Coefficient of thermal conductivity	Heat energy × Thickness Area × Temperature × Time	[ML ² T ⁻²][L] [L ²][K][T]	[MLT ⁻³ K ⁻¹]	W m ⁻¹ K
52.	Bulk modulus	Volume ×(Change in pressure) Change in volume	$\frac{[L^3][ML^{-1}T^{-2}]}{[L^3]}$	[ML ⁻¹ T ⁻²]	N m ⁻²
53.	Centripetal acceleration	(Velocity) ² Radius	$\frac{[LT^{-1}]^2}{[L]}$	[M [•] LT ⁻²]	m s ⁻²
54.	Stefan's constant	Energy (Area) × (time) × (temperature) ⁴	$\frac{[ML^2T^{-2}]}{[L^2][T][K]^4}$	[ML ⁰ T ⁻³ K ⁻⁴]	W m ⁻² K-
55.	Wien's constant	Wavelength × Temperature	[L][K]	[MºLTºK]	m K

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5 6.	Universal gas constant	Pressure × Volume Mole × Temperature	$\frac{[ML^{-2}T^{-2}][L^3]}{[mol][K]}$	[ML ² T ⁻² K ⁻¹ mol ⁻¹]	J K_ ¹ mol ⁻¹
57.	Boltzmann constant	Universal gas constant Avogadro's number	$\frac{[ML^2T^{-2}K^{-1}mol^{-1}]}{[mol^{-1}]}$	[ML ² T ^{.2} K ⁻¹]	J K ⁻¹
58.	Charge	Current × Time	[A][T]	[M ^u L•TA]	C (coulomb)
59.	Current density	<u>Current</u> Area	$\frac{[A]}{[L^2]}$	[M ⁰ L- ² T ⁰ A]	A m ⁻²
60.	Electric potential, electromotive force, voltage	Work Charge	$\begin{bmatrix} \underline{ML^2T^{-2}} \\ I \end{bmatrix}$	[MIL ² T ⁻³ A ⁻¹]	V (volt)
61.	Resistance	Potential difference Current	$\begin{bmatrix} ML^{2}T^{-3}A^{-1} \\ \hline \\ [A] \end{bmatrix}$	[ML ² T ⁻³ A ⁻²]	Ω (ohm)
62.	Capacitance	Charge Potential difference	$\frac{[AT]}{[ML^2T^{-3}A^{-1}]}$	$[M^{-1}L^{-2}T^{4}A^{2}]$	F (farad)
63.	Electric field	Electrical force Charge	[<u>MLT⁻²]</u> [AT]	[ML .T ⁻³ A ⁻¹]	N C-1
64	Electric flux	Electric field × Area	[MLT-3A-1][L ²]	[<u>MT</u>] ³ T ⁻³ A ⁻¹]	N m ² C ⁻¹
65.	Electric dipole	Charge × Length	[AT][L]	[M ⁰ LTA]	Cm
66.	Electric field strength or electric intensity	Potential difference Distance	$\left \frac{[ML^2T^{-3}A^{-1}]}{[L]} \right $	[MLT ³ A ⁻¹]	V m-1
67.	Magnetic field, magnetic flux density, nagnetic induction	Force Current × Length	[MLT- ²] [A][L]	[[ML ⁰ T ⁻² A ⁻¹]	T (tesla)
68.	Magnetic flux	Magnetic field × Area	[ML ⁰ T ⁻² A ⁻¹][L ²]	$[ML^2T^{-2}A^{-1}]$	Wb (weber)
69.	Inductance	Magnetic flux Current	$\frac{[ML^2T^{-2}A^{-1}]}{[A]}$	[ML ² T ⁻² A ⁻²]	H (henry)
70.	Magnetic dipole moment	Current × Area	$\left[\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} L^2 \end{bmatrix} \right]$	[M [#] L ² T ⁰ A]	A m ²
71.	Intensity of magnetisation	Magnetic moment Volume	[L ² A] [L ³]	[M [•] L ⁻¹ T [•] A]	A m ⁻¹
72.	Pemuttivity of free space	$\frac{\texttt{Charge} \times \text{Charge}}{4\pi \times \text{Electric force} \times (\text{Distance})^2}$	$\frac{[\Delta T][AT]}{[MLT^{-2}][L]^2}$	[M ⁻¹ L ⁻³ T ⁴ A ²]	$C^2 N^{-1} m^{-2}$
73.	Permeability of free space	2π × Force × Distance Current × Current × Length	$\frac{[MLT^{-2}][L]}{[A][A][L]}$	[MLT-2A-3]	T m A ⁻¹
74.	Refractive index	Speed of light in vacuum Speed of light in medium	$ \left \begin{array}{c} \frac{[LT^{-1}]}{[LT^{-1}]} \end{array} \right $	[M ⁰ T ⁰ T ⁰]	No units

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75.	Faraday constant	Avogadro's number × Elementary charge	[mol ⁻¹][AT]	[M ⁰ L ⁰ TAmol ⁻¹]	C mol ⁻¹
76.	Wave number	2π Wavelength	1 [L]	[M ⁰ L ⁻¹ T ⁰]	rad m ⁻¹
77.	Radiant flux, radiant power	Energy emitted Time	$\frac{[ML^2T^{-2}]}{[T]}$	[ML ² T-3]	watt
78.	Luminosity of radiant flux or radiant intensity	Radiant power or <u>Radiant flux of source</u> Solid angle	[MI ² T ⁻³] [M ⁰ L ⁰ T ⁰]	[ML ² T ⁻³]	watt sr ⁻¹
79.	Luminous power or luminous flux of source	Luminous energy emitted Time	$\frac{[ML^2T^{-2}]}{[T]}$	[ML ² T-3]	lumen
80.	Relative luminosity	Luminous flux of a source of given wavelenth Luminous flux of peak sensitivity wavelength (555 nm) source of same power	$\frac{[ML^2T^{-3}]}{[ML^2L^{-3}]}$	[MºLºTº]	No units
81.	Luminous efficiency	Total luminous flux Total radiant flux	$\frac{[ML^2T^{-3}]}{[ML^2T^{-3}]}$	[M ⁰ L ⁰ T ⁰]	N o units
82.	Illuminance or illumination	Luminous flux incident Area	$\frac{[ML^2T^{-3}]}{[L^2]}$	[ML ⁰ 'I ⁻³]	lux
83.	Decay constant	0.693 Half life	[T-1]	[M ⁰ L ⁰ T ⁻¹]	s ⁻¹
84.	Resonant frequency	$\frac{1}{2\pi\sqrt{\text{Inductance}} \times \text{Capacitance}}$	$[ML^{2}T^{-2}A^{-2}]^{-1/2} \times [M^{-1}L^{-2}T^{4}A^{2}]^{-1/2}$	[M ⁰ L ⁰ A ⁰ T ⁻¹]	hertz (Hz)
85.	Quality factor or Q-factor of coil	Resonant frequency <u>× Inductance</u> Resistance	$\frac{[T^{-1}][ML^{2}T^{-2}A^{-2}]}{[ML^{2}T^{-3}A^{-2}]}$	[M ⁰ L ⁰ T ⁰]	No units
86.	Power of lens	(Focal length) ⁻¹	[L] ⁻¹	$[M^0L^{-1}T^0]$	dioptre (D)
87.	Magnification	Size of image Size of object		[M ⁰ L ⁰ T ⁰]	No units
88.	Fluid flow rate	$\frac{(\pi/8)(\text{Pressure}) \times (\text{Radius})^4}{(\text{Viscosity coefficient}) \times (\text{Length})}$	$\frac{[ML^{-1}T^{-2}][L]^4}{[ML^{-1}T^{-1}][L]}$	[M ⁰ L ³ T ⁻¹]	m ³ s ⁻¹
89.	Capacitive reactance	(Angular frequency × Capacitance) ⁻¹	$[T^{-1}]^{-1}[M^{-1}L^{-2}T^4A^2]^{-1}$	[ML ² T ⁻³ A ⁻²]	ohm (Ω)
90.	Inductive reactance	(Angular frequency × Inductance)	[T ⁻¹][ML ² T ⁻² A ⁻²]	[ML ² T ⁻³ A ⁻²]	ohm (Ω)

Here it is worthy to note that constants such as $\frac{1}{2}$, π or trigonometrical functions such as $\sin\omega t$, (ratio of two sides), etc. have no units and dimensions.

There are various physical quantities which have same dimensional formula. The list of such physical quantities and corresponding dimensional formula are given below.

S. No.	Physical Quantities	Dimensional Formula
1.	Frequency, angular frequency, angular velocity, velocity gradient	[M ⁰ L ⁰ T ⁻¹]
2.	Work, internal energy, potential energy, minetic energy, torque, moment of force	[ML ² T ⁻²]
3.	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density	[ML ⁻¹ T ⁻²]
4.	Momentum and impulse	[MLT ⁻¹]
5.	Acceleration, Acceleration due to gravity, gravitational field intensity	[M ⁰ LT ⁻²]
6.	Thrust, force, weight, energy gradient	[MLT ⁻²]
7.	Angular momentum and Planck's constant (h)	[ML ² T ⁻¹]
8.	Surface tension, force gradient, spring constant	[ML ⁰ T ⁻²]
9.	Strain, refractive index, relative density, angle, solid angle, distance gradient relative permeability, relative permittivity	[M ⁰ L ⁰ T ⁰]
10.	If P is pressure, V is volume, m is mass, s is specific heat, L is latent heat, ΔT is rise in temperature then PV., mL, (ms ΔT) all have dimensions of energy	[ML ² T ⁻²]
11.	If <i>l</i> is length, <i>g</i> is acceleration due to gravity, <i>m</i> is mass, <i>k</i> is force constant, <i>R</i> is radius of earth, then $\left(\frac{1}{g}\right)^{1/2}, \left(\frac{m}{k}\right)^{1/2}, \left(\frac{R}{g}\right)^{1/2}$ all have the dimensions of time.	[M [●] L ⁰ T]
12.	If <i>L</i> is inductance, <i>R</i> is resistance, <i>C</i> is capacitance then <i>L/R</i> , <i>CR</i> and \sqrt{LC} all have the dimensions of time.	[MºLºT]

Illustration 3

If a composite physical quantity in terms of moment of inertia I, force F, velocity v, work W and length L is defined as,

$$Q = \left(\frac{IFv^2}{WL^3}\right)$$

Find the dimensions of Q and identify it. **Soln.** : As $[I] = [ML^2], [F] = [MLT^{-2}]$ $[v] = [LT^{-1}]$ and $[W] = [ML^2T^{-2}]$

$$\begin{split} [Q] = \frac{[ML^2][MLT^{-2}][LT^{-1}]^2}{[ML^2T^{-2}][L]} \cdot \\ [Q] = [MT^{-2}] \end{split}$$

As [MT-2] are dimensions of surface tension, force constant or surface energy, i.e., energy per unit area, the physical quantity may be any one of these.

APPLICATIONS OF DIMENSIONAL ANALYSIS

(1) To check the dimensional correctness of a given physical relation.

This is based on the 'principle of homogeneity of dimensions'. According to this principle the dimensions of each term on both sides of an equation must be the same i.e., [LHS] = [RHS].

This principle is based on the fact that only quantities of same kind can be added or subtracted. For example, if we have an equation like A = B + C, the quantities A, B and C must have the same dimensions. A dimensionally correct equation may or may not be physically correct.

Example

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(i) Consider the formula

$$S = ut - \left(\frac{1}{4}\right)at^2$$

Dimensionally, $[L] = [LT^{-1}][T] - [LT^{-2}][T^2]$ *i.e.*, [L] = [L] - [L]

As in the above equation dimensions of each term on both sides are same, so this equation is dimensionally correct. However, from equations of motion we know that

$$S = ut + \left(\frac{1}{2}\right)at^2$$

So the given equation is physically wrong though it is correct dimensionally.

(ii) Consider the formula,

$$F = \frac{mv^2}{r^2}$$

Dimensionally, [MLT⁻²] = $\frac{[M][LT^{-1}]^2}{[L]^2}$

 $[MLT^{-2}] = [MT^{-2}]$ i.e.

As in the above equation dimensions on both sides are not same; this formula is not correct dimensionally, so can never be correct physically.

(2) To convert a physical quantity from one system of units to the other.

This is based on the fact that magnitude of a physical quantity remains same whatever system is used for measurement *i.e.* magnitude = numerical value $(n) \times unit (u) = constant$

or
$$n_1 u_1 = n_2 u_2$$

so if a quantity is represented by [M^eL^bT^c] then

$$n_2 = n_1 \frac{u_1}{u_2} = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^b$$

Example

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(i) Pressure is given by $P = \frac{F}{A}$ Thus dimensional formula of pressure is

$$[P] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$