In SI units, 1 pascal = (1 kg) $(1\ \text{m})^{-1}$ $(1\ \text{s})^{-2}$

In CGS units, 1 CGS pressure = (1 g) $(1\ \text{cm})^{-1}$ $(1\ \text{s})^{-2}$

$$\text{Thus, } \frac{1\ \text{pascal}}{1\ \text{CGS pressure}} = \left(\frac{1\ \text{kg}}{1\ \text{g}}\right)\left(\frac{1\ \text{m}}{1\ \text{cm}}\right)^{-1}\left(\frac{1\ \text{s}}{1\ \text{s}}\right)^{-2}$$

$$= (10^3)(10^2)^{-1} = 10$$

or 1 pascal = 10 CGS pressure

(ii) The newton is the SI unit of force and has dimensional formula $[MLT^{-2}]$.

So, 1 newton = 1 kg m s^{-2}

but 1 kg = 10^3g and 1 m = 10^2 cm

$$\text{So, } 1\ \text{N} = \frac{(10^3\ \text{g})}{\text{s}^2}(10^2\ \text{cm}) = 10^5\ \frac{\text{g cm}}{\text{s}^2} = 10^5\ \text{dyne}$$

3. Deducing relationship among the physical quantities

If one knows the quantities on which a particular physical quantity depends and if one guesses that this dependence is of product type, method of dimension may be helpful in the derivation of the relation.

Example

The time period of a simple pendulum depends upon (i) mass m of the bob (ii) length l of the string and (iii) acceleration due to gravity g.

Hence, $T = K\, m^a l^b g^c$

where K is a dimensionless constant and a, b and c are the unknown numbers.

Taking dimension of both sides, we get

$[T] = [M]^a [L]^b [LT^{-2}]^c$

$\Rightarrow [M^0L^0T^1] = [M^a][L^{b+c}][T^{-2c}]$

Equating the dimensions of M, L and T on both sides, we get

$a = 0;\ b + c = 0$ and $-2c = 1$

$\therefore\ a = 0,\ b = \frac{1}{2}$ and $c = -\frac{1}{2}$

Putting the values of a, b, c in equation (i), we get

$$T = K\sqrt{\frac{l}{g}}$$

It is found that $K = 2\pi$

$$\therefore\ \text{Time period} = T = 2\pi\sqrt{\frac{l}{g}}$$

(4) To find the dimensions of constants in a relation

Sometimes a physical relation contains constants. By using the dimensional formula of the various physical quantities, we evaluate the constant.

Example

(i) For Newton's law of gravitation, we have

$$F = G\frac{m_1m_2}{r^2} \quad \text{or} \quad G = \frac{Fr^2}{m_1m_2}$$

$$\therefore\ [G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{[MLT^{-2}][L^2]}{[M][M]}$$

or $[G] = [M^{-1}L^3T^{-2}]$

(ii) According to Planck,

$$E = h\upsilon \quad \text{or} \quad h = \frac{E}{\upsilon} \quad \therefore\ [h] = \frac{[E]}{[\upsilon]} = \frac{[ML^2T^{-2}]}{[T^{-1}]}$$

or $[h] = [ML^2T^{-1}]$

Limitations of Theory of Dimensions

- If dimensions are given, physical quantity may not be unique as many physical quantities have same dimensions. For example if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$ it may be work or energy or torque.
- Numerical constants $[K]$ having no dimensions such as $\left(\frac{1}{2}\right)$, 1 or 2π etc, cannot be deduced by the method of dimensions.
- The method of dimensions cannot be used to derive relations other than product of power functions. For example,

$$S = ut + \frac{1}{2}at^2 \quad \text{or} \quad y = a\sin\omega t$$

  cannot be derived by using this theory.
- The method of dimensions cannot be applied to derive formula if in mechanics a physical quantity depends on more than 3 physical quantities as then there will be less number (= 3) of equations than the unknowns (> 3).

ERRORS IN MEASUREMENTS

The difference between the true value and the measured value of a quantity is known as the error of measurement.

Classification of Errors

Errors may arise from different sources and are usually classified as follows :

(1) Systematic or controllable errors : Systematic errors are the errors whose causes are known. They can be either positive or negative. Due to known causes these errors can be minimised. Systematic errors can further be classified into three categories.

- **Instrumental errors :** These errors are due to imperfect design or erroneous manufacture or misuse of the measuring instrument. These can be reduced by using more accurate instruments.
- **Environmental errors :** These errors are due to the changes in external environmental conditions such as temperature, pressure, humidity, dust, vibrations or magnetic and electrostatic fields.
- **Observational errors :** These errors arise due to improper setting of the apparatus or carelessness in taking observations.

(2) Random errors : These errors are due to unknown causes. Therefore they occur irregularly and are variable in magnitude and sign. Since the causes of these errors are not known precisely they cannot be eliminated completely. For example, when the same person repeats the same observation in the same conditions, he may get different readings different times.

Random errors can be reduced by repeating the observation a large number of times and taking the arithmatic mean of all the observations. This mean value would be very close to the most accurate reading.

For example, if the random error in the arithmatic mean of 100 observations is 'X' then the random error in the arithmatic mean of 500 observations will be $\frac{X}{5}$.

(3) Gross errors : Gross errors arise due to human carelessness and mistakes in reading the instruments or calculating and recording the measurement results.

For example –

- Reading instrument without proper initial settings.
- Taking the observations wrongly without taking necessary precautions.
- Exhibiting mistakes in recording the observations.
- Putting improper values of the observations in calculations.

These errors can be minimised by increasing the sincerity and alertness of the observer.

Representation of Errors

Errors can be expressed in the following ways

- **Absolute error :** The difference in the magnitude of the true value and the measured value of a physical quantity is called absolute error.

  If a_1, a_2, a_3 ...., a_n are n different readings of a physical quantity in an experiment, then true value $(\bar{a})$ of the quantity is

  $$\bar{a} = \frac{a_1 + a_2 + a_3 + \ldots\ldots + a_n}{n}$$

  The absolute errors in the various measured values are

  $$\Delta a_1 = \bar{a} - a_1;\ \Delta a_2 = \bar{a} - a_2,\ \ldots\ldots\ldots,\ \Delta a_n = \bar{a} - a_n$$

- **Mean absolute error :** The arithmetic mean of all absolute errors in the measured values is called mean absolute error.

  $$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots\ldots + |\Delta a_n|}{n}$$

- **Relative error and percentage error :** The ratio of mean absolute error to the mean value (or true value) of the quantity being measured is called relative error or fractional error.

  $$\text{Relative error} = \frac{\overline{\Delta a}}{\bar{a}}$$

  $$\therefore\ \text{Percentage error} = \frac{\overline{\Delta a}}{\bar{a}} \times 100$$

Illustration 4

The period of oscillation of a simple pendulum in an experiment is recorded as 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. Find (i) mean time period (ii) absolute error in each observation and percentage error.

Soln. : (i) Mean time period is given by

$$\bar{T} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5} = \frac{13.12}{5} = 2.62\text{ s}$$

(ii) The absolute error in each observation is

2.62 – 2.63 = –0.01, 2.62 – 2.56 = 0.06, 2.62 – 2.42 = 0.20, 2.62 – 2.71 = –0.09, 2.62 – 2.80 = – 0.18

$$\text{Mean absolute error, } \overline{\Delta T} = \frac{\Sigma|\Delta T|}{5}$$

$$= \frac{0.01 + 0.06 + 0.2 + 0.09 + 0.18}{5} = 0.11\text{ sec}$$

$$\therefore\ \text{Percentage error} = \frac{\overline{\Delta T}}{\bar{T}} \times 100 = \frac{0.11}{2.62} \times 100 = 4.2\%$$

Propagation of Errors in Mathematical Operations

- **Error in sum or difference :** The maximum absolute error in the sum or difference of the two quantities is equal to the sum of the absolute errors in the individual quantities. If $X = A + B$ or $A - B$ and if $\pm\Delta A$ and $\pm\Delta B$ represent the absolute errors in A and B respectively, then the maximum absolute error in $X = \Delta X = \Delta A + \Delta B$ and

  $$\text{Maximum percentage error} = \frac{\Delta X}{X} \times 100$$

  The result will be written as $X \pm \Delta X$ (in terms of absolute error)

  or $X \pm \frac{\Delta X}{X} \times 100$ (in terms of percentage error)

- **Error in product or division :** The maximum fractional or relative error in the product or division of quantities is equal to the sum of the fractional or relative errors in the individual quantities.

  If $X = A \times B$ or $X = A/B$

  $$\text{then } \frac{\Delta X}{X} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

- **Error in power of a quantity :** The maximum fractional error in a quantity raised to a power (n) is n times the fractional error in the quantity itself, i.e.

  $$\text{If } X = A^n \text{ then } \frac{\Delta X}{X} = n\left(\frac{\Delta A}{A}\right)$$

- **General case :**

  $$\text{If } X = A^p B^q C^r \text{ then } \frac{\Delta X}{X} = \left[p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)\right]$$

  $$\text{If } X = \frac{A^p B^q}{C^r} \text{ then } \frac{\Delta X}{X} = \left[p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)\right]$$

Illustration 5

If two lengths a and b are given as :

$a = 25.4$ cm ± 0.1 cm and $b = 16.5$ cm ± 0.1 cm

Then find $a + b$.

Soln.: Let $Q = a + b$

Average value of $Q = 25.4 + 16.5 = 41.9$ cm

Also, maximum value of $Q = (25.4 + 0.1) + (16.5 + 0.4)$
$= 42.4$ cm

∴ Magnitude of maximum error in Q
$= 42.4 - 41.9 = 0.5$ cm

$Q = a + b = (41.9 \pm 0.5)$ cm

Illustration 6

A body travels uniformly a distance (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Calculate its velocity with error limits. What is the percentage error in velocity?

Soln. : Given distance, $s = (13.8 \pm 0.2)$ m

and time $t = (4.0 \pm 0.3)$ s

$$\text{Velocity } v = \frac{s}{t} = \frac{13.8}{4.0} = 3.45\text{ m s}^{-1} = 3.5\text{ m s}^{-1}$$

$$\frac{\Delta v}{v} = \pm\left(\frac{\Delta s}{s} + \frac{\Delta t}{t}\right) = \pm\left(\frac{0.2}{13.8} + \frac{0.3}{4.0}\right)$$

$$= \pm\left(\frac{0.8 + 4.14}{13.8 \times 4.0}\right) = \pm\frac{4.49}{13.8 \times 4.0} = \pm 0.0895$$

$\therefore\ \Delta v = \pm 0.0895 \times V = \pm 0.0895 \times 3.45 = \pm 0.3087 = \pm 0.31$

Hence $v = (3.5 \pm 0.31)$ m s^{-1}

$$\text{Percentage error in velocity} = \frac{\Delta v}{v} \times 100 = \pm 0.0895 \times 100$$

$$= \pm 8.95\% = \pm 9\%$$

Illustration 7

A thin copper wire of length L increases in length by 2% when heated from T_1 to T_2. If a copper cube having side $10L$ is heated from T_1 to T_2, what will be the percentage change in
(i) area of one face of the cube and
(ii) volume of the cube

Soln. : (i) Area $A = 10L \times 10L = 100L^2$

Percentage change in area

$$= \frac{\Delta A}{A} \times 100 = 2 \times \frac{\Delta L}{L} \times 100 = 2 \times 2\% = 4\%$$

$$\left(\because \ \frac{\Delta A}{A} = \frac{\Delta 100}{100} + 2\frac{\Delta L}{L} = 0 + 2\frac{\Delta L}{L} = 2\frac{\Delta L}{L}\right)$$

(ii) Volume $V = 10L \times 10L \times 10L = 1000L^3$

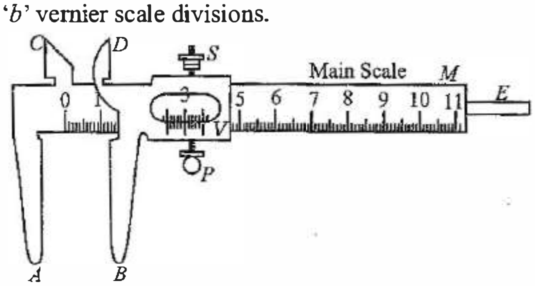
Percentage change in volume

$$= \frac{\Delta V}{V} \times 100 = 3\frac{\Delta L}{L} = 3 \times 2\% = 6\%$$

LEAST COUNT

The smallest value of a physical quantity which can be measured accurately with an instrument is called the least count (L.C) of the measuring instrument.

(i) **Least count of vernier callipers** – Suppose the size of one main scale division (M.S.D) is M units and that of one vernier scale division (V.S.D) is V units. Also let the length of 'a' main scale divisions is equal to the length of 'b' vernier scale divisions.



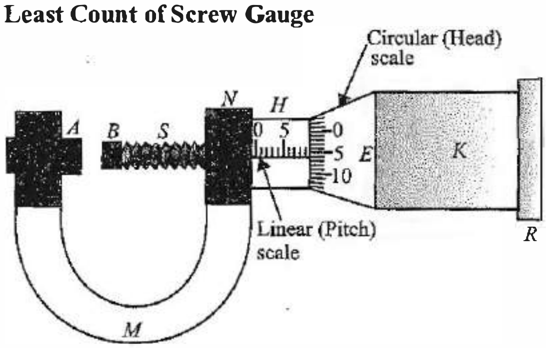
Therefore $aM = bV$ or $V = \frac{a}{b}M$

Therefore $M - V = M - \frac{a}{b}M$ or $M - V = \left(\frac{b-a}{b}\right)M$

The quantity $(M - V)$ is called vernier constant (V.C) or least count (L.C) of the vernier callipers.

$$L.C. = \left(\frac{b-a}{b}\right)M$$

(ii) **Least Count of Screw Gauge**



$$\text{Least count} = \frac{\text{Pitch}}{\text{Number of divisions on the circular scale}}$$

where pitch is defined as the distance moved by the screw head when the circular scale is given one complete rotation i.e.

$$\text{Pitch} = \frac{\text{Distance moved by the screw on the linear scale}}{\text{Number of full rotations given}}$$

Illustration 8

The smallest division on main scale of a vernier callipers is 1 mm and 10 vernier divisions coincide with 9 scale divisions. While measuring the length of a line, the zero mark of the vernier scale lies between 10.2 cm and 10.3 cm and the third division of vernier scale coincide with a main scale division.
(a) Determine the least count of the callipers
(b) Find the length of the line

Soln.: (a) Least count (L.C)

$$= \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$$

$$= \frac{1}{10}\text{ mm} = 0.1\text{ mm} = 0.01\text{ cm}$$

(b) Length of the line = $(10.2 + 3 \times 0.01)$ cm = 10.23 cm

Illustration 9

The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. In measuring the diameter of a sphere there are six divisions on the linear scale and forty divisions on circular scale coincides with the reference line. Find the diameter of the sphere.

Soln.: $\text{L.C.} = \frac{1}{100} = 0.01\text{ mm}$

Linear scale reading = 6(pitch) = 6 mm

Circular scale reading = $40 \times 0.01 = 0.4$ mm

$\therefore$ Total reading = $(6 + 0.4) = 6.4$ mm

Accuracy and Precision in Measurements

- The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity while precision tells us to what resolution or limit the quantity is measured by a measuring instrument. Precision is determined by the least count of the measuring instrument. Smaller the least count, greater is the precision.

  For example, suppose true value of certain length = 3.678 cm

  In 1st experiment, resolution = 0.1 cm
  measured value = 3.5 cm

  In 2nd experiment, resolution = 0.01 cm
  measured value = 3.38 cm

  Here 1st measurement has more accuracy and less precision while 2nd measurement has less accuracy but more precise.

SIGNIFICANT FIGURES

- Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. "All accurately known digits in a measurement plus the first uncertain digit together form significant figures"

Rules to Find the Significant Figures

- **Rule-1:**
  All non-zero digits are significant. *e.g.* 1324 has four significant figures.

- **Rule-2 :**
  All zeros occuring between two non-zero digits are significant. *e.g.* 120024 has 6 significant digits.
- **Rule-3 :**
  If the number is less than 1, the zero(s) on the right of decimal point is significant, but to the left of the first non-zero digit are not significant. *e.g.* 0.00064 has two significant digits.
- **Rule-4 :**
  In a number without a decimal point the terminal or trailing zero(s) are not significant. *e.g.* 227800 have four significant digits.
- **Rule-5 :**
  In a number with a decimal point the trailing zero(s) are significant. *e.g.* 3.200 or 0.05400 have four significant digits each.
  Note : The power (or exponent) of 10 is irrelevant to the determination of significant figures. For example, 3.100×10^2 has 4 significant figures.

Rounding off

- **Rule-1:**
  If the digit to be dropped is less than 5, then the preceding digit is left unchanged.
  e.g. 8.22 is romded off to 8.2.
- **Rule-2:**
  If the digit to be dropped is more than 5, then the preceding digit is raised by one.
  e.g. $x = 6.87$ is rounded off to 6.9.
- **Rule-3:**
  If the digit to be dropped is 5 followed by digit other than zero, then the preceding digit is raised by one.
  e.g. 7.851 is rounded off to 7.9.
- **Rule-4:**
  If the digit to be dropped is 5 or 5 followed by zero, then preceding digit is left unchanged, if it is even.
  e.g. 5.250 is rounded off to 5.2.
- **Rule-5:**
  If the digit to be dropped is 5 or 5 followed by zero, then the preceding digit is raised by one, if it is odd.
  e.g. 3.750 is rounded off to 3.8.

Rules for Arithmetic Operation with Significant Figures

- In both, addition or subtraction the final result should retain as many decimal places as are there in the number with least decimal places.
  e.g. $24.36 + 0.0623 + 256.2 = 280.6223$
  The result should be rounded off to 280.6
- In multiplication or division, the final result should retain as many significant figures as there in the original number with the least significant figures.
  e.g. $4.6 \times 0.128 = 0.5888$
  The result should be rounded off to 0.59

Illustration 10

Write down the number of significant figures in the following
(a) 6928 (b) 62.00 m (c) 0.00625 cm (d) 1200 N

Soln. : (a) 4 (b) 4 (c) 3 (d) 2

Illustration 11

Round off to four significant figures :
(a) 45.689 (b) 2.0082

Soln.: (a) 45.69 (b) 2.008

Illustration 12

A thin wire has a length of 21.7 cm and radius 0.46 mm. Calculate the volume of the wire to correct significant figures.

Soln.: Given $l = 21.7$ cm, $r = 0.46$ mm $= 0.046$ cm

Volume of wire $V = \pi r^2 l$

$$= \frac{22}{7}(0.046)^2(21.7) = 0.1443\text{cm}^2 = 0.14\text{cm}^3$$

The result is rounded off to least number of significant figures in the given measurements i.e., 2(in 0.46 mm).