If a particle travels a distance S in time t_1 to t_2, the average speed is $v_{av} = \frac{S}{t_2 - t_1}$

If a particle travels a distance s_1, s_2, s_3 etc. with speeds v_1, v_2, v_3 etc. respectively, then total travelled distance $S = s_1 + s_2 + s_3 + ... + s_n$

Total time taken of trip $= \frac{s_1}{v_1} + \frac{s_2}{v_2} + ... + \frac{s_n}{v_n}$

Average speed of a trip $= \frac{s_1 + s_2 + s_3 + ... + s_n}{\left(\frac{s_1}{v_1} + \frac{s_2}{v_2} + ... + \frac{s_n}{v_n}\right)}$

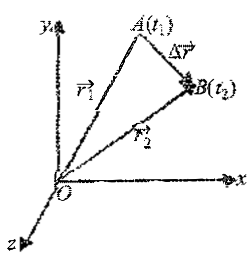
VELOCITY

- Velocity of a body is defined as the rate of change of displacement of the body with time

  i.e., Velocity $= \frac{\text{displacement}}{\text{time interval}}$
- Speed is a scalar quantity whereas velocity is a vector quantity.
- Both the speed and velocity have the same unit and same dimensional formula $[M^0LT^{-1}]$.
- **Average velocity :** Average velocity is defined as the ratio of the displacement to the time interval for which the motion takes place.

  i.e., Average velocity $= \frac{\text{displacement}}{\text{time taken}}$

  Let a particle be at point A at time t_1 and point B at time t_2. Position vectors of A and B are $\vec{r}_1$ and $\vec{r}_2$ respectively. The displacement in this time interval is the vector $\overrightarrow{AB} = (\vec{r}_2 - \vec{r}_1)$. The average velocity in this time interval is



$$\vec{v}_{av} = \frac{\overrightarrow{AB}}{t_2 - t_1} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

Here $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_2 - \vec{r}_1$ = change in position vector.

For small time interval between t and $t + \Delta t$, change in position is $\Delta\vec{r}$ then average velocity in Δt time interval is $\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$.

- **Instantaneous velocity :** The velocity of a body at a given instant of time during motion is known as instantaneous velocity *i.e.*,

  Instantaneous velocity $= \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$
- The magnitude of instantaneous velocity is equal to the instantaneous speed at the given instant.
- The speedometer of an automobile measures the instantaneous speed of the automobile.
- The average speed of a body is greater or equal to the magnitude of the average velocity over a given time interval.

ACCELERATION

- Acceleration of a body is defined as the rate of change of velocity of a body with time.

  i.e. Acceleration $= \frac{\text{change in velocity}}{\text{time taken}}$
- **Average acceleration :** Average acceleration is defined as the ratio of the change in velocity to the time interval during which the change occurs

  i.e. Average acceleration $= \frac{\text{change in velocity}}{\text{time interval}}$
- **Instantaneous acceleration :** The acceleration of a body at a given instant of time is known as instantaneous acceleration

  i.e. Average acceleration $= \frac{\text{change in velocity}}{\text{time interval}}$
- If a body is speeding up, acceleration is in the direction of velocity, if its speed is decreasing, acceleration is in the direction opposite to that of the velocity. This statement is independent of the choice of the origin and the axis.
- The zero velocity of a body at any instant does not necessarily imply zero acceleration at that instant. A body may be momentarily at rest and yet have non-zero acceleration. For example, a body thrown up has zero velocity at its uppermost point but the acceleration at that instant continues to be the acceleration due to gravity.

Illustration 1

The relation between time t and displacement x is $t = \alpha x^2 + \beta x$, where α and β are constants, find the relation between velocity and acceleration.

Soln. : We have the relation between time t and displacement x as

$$t = \alpha x^2 + \beta x$$

Differentiating with respect to x, we get, $\frac{dt}{dx} = 2\alpha x + \beta$

$$\therefore \quad \frac{dx}{dt} = \frac{1}{2\alpha x + \beta} \quad \text{or} \quad v = \frac{1}{2\alpha x + \beta} \quad \left[\because \frac{dx}{dt} = \text{velocity } v\right]$$

Now, acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{1}{2\alpha x + \beta}\right]$

$$\Rightarrow \frac{d}{dx}\left[\frac{1}{2\alpha x + \beta}\right] \cdot \frac{dx}{dt} = -\frac{2\alpha}{(2\alpha x + \beta)^2} \cdot v = -2\alpha v^2 . v = -2\alpha v^3.$$

$$\Rightarrow \quad a = -2\alpha v^3.$$

Illustration 2

A body moving in a straight line with uniform acceleration describes three successive equal distances in time intervals t_1, t_2 and t_3 respectively. Show that

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$$

Soln. : Let v_1, v_2 and v_3 be the initial velocities of the particle in the time intervals t_1, t_2 and t_3 respectively, and v be the final velocity of the particle in the time interval t_3. The particle moves equal distances in each time interval and let it be d.

Then, average velocity in the time interval t_1 is

$$\frac{d}{t_1}=\frac{1}{2}(v_1+v_2) \qquad \text{...(i)}$$

Similarly, $$\frac{d}{t_2}=\frac{1}{2}(v_2+v_3) \qquad \text{...(ii)}$$

$$\frac{d}{t_3}=\frac{1}{2}(v_3+v) \qquad \text{...(iii)}$$

and average velocity in the time interval $(t_1+t_2+t_3)$ is

$$\frac{3d}{t_1+t_2+t_3}=\frac{1}{2}(v_1+v) \qquad \text{...(iv)}$$

From eqn (i), (ii), (iii) and (iv) we get,

$$\frac{1}{t_1}-\frac{1}{t_2}+\frac{1}{t_3}=\frac{1}{2d}(v_1+v_2)-\frac{1}{2d}(v_2+v_3)+\frac{1}{2d}(v_3+v)$$

$$=\frac{1}{2d}(v_1+v)=\frac{3}{t_1+t_2+t_3} \quad \text{(Proved).}$$

Illustration 3

At time t, positions of three particles A, B and C are as follows:

$x_A=2t+7,\ x_B=3t^2+2t+6,\ x_C=5t^3+4t$

Which of them has uniform (constant) acceleration?

Soln. : (a) $v=\frac{dx_A}{dt}=2$ and $a=\frac{d^2x_A}{dt^2}=0$

The particle has no acceleration at all.

(b) $v=\frac{dx_B}{dt}=(3\times 2t)+2+0=6t+2$

$a=\frac{d^2x_B}{dt^2}=6$

Here, acceleration is uniform.

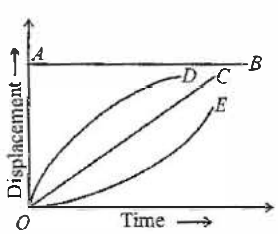
(c) $v=\frac{dx_C}{dt}=5\times 3t^2+4=15t^2+4$

$a=\frac{dv}{dt}=\frac{d^2x_C}{dt^2}=15\times 2t=30t.$

Here acceleration depends upon time, so it is not uniform.

DISPLACEMENT-TIME GRAPH

(i) If the graph is a straight line parallel to time-axis, shown by line AB, it means that the body is at rest *i.e.* velocity = zero.



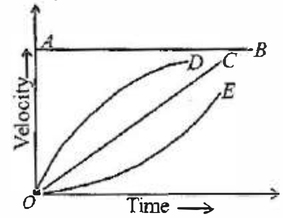
(ii) If the graph is a straight line inclined to time-axis (such as OC) shows that body is moving with a constant velocity.

(iii) If the graph obtained is a curve like OD whose slope decreases with time, the velocity goes on decreasing, *i.e.*, motion is retarded.

(iv) If the graph obtained is a curve like OE whose slope increases with time, the velocity goes on increasing, *i.e.* motion is accelerated.

VELOCITY-TIME GRAPH

(i) If the graph is a straight line parallel to time axis shown by line AB, it means that the body is moving with a constant velocity or acceleration (a) is zero.



(ii) If the graph is a straight line and inclined to the time-axis with +ve slope (line OC) it means that the body is moving with constant acceleration.

(iii) If the graph obtained is a curve like OD whose slope decreases with time, the acceleration goes on decreasing.

(iv) If the graph obtained is a curve like OE whose slope increases with time, the acceleration goes on increasing.

(v) The area of velocity-time graph with time axis represents the displacement of that body.

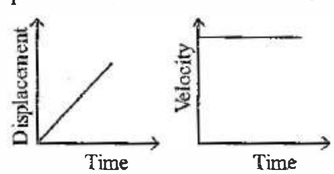
ACCELERATION-TIME GRAPH

(i) When the graph is a straight line and parallel to time axis then acceleration is constant.

(ii) When the graph is oblique straight line having positive slope, then acceleration is uniformly increasing.

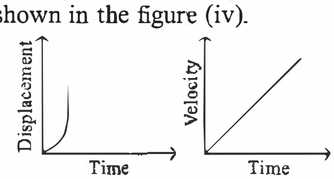
(iii) When the graph is an oblique straight line having negative slope, then acceleration is uniformly decreasing.

- For uniform motion, acceleration is zero, displacement-time graph is a straight line inclined to the time axis as shown in the figure (i) and velocity time graph is a straight line parallel to time axis as shown in figure (ii).



(i) (ii)

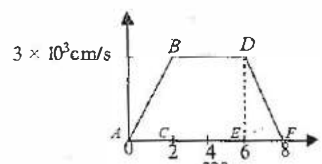
- For motion with uniform acceleration, displacement-time graph is a parabola as shown in figure (iii) while velocity-time graph is a straight line inclined to time axis as shown in the figure (iv).



(iii) (iv)

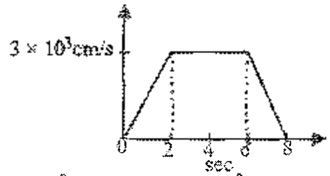
Illustration 4

The velocity,-time graph of the motion of a car is given below. Find the distance travelled by the car in the first six seconds. What is the deceleration of the car during the last two seconds?



Soln.: The area enclosed between v-t curve gives the distance travelled by the body.

$\therefore$ Distance travelled by the car in the first six seconds = area of triangle ABC + area of rectangle $BDEC$



$$= \frac{1}{2} \times 2 \times 3 \times 10^3 + (6-2) \times 3 \times 10^3$$

$= 3 \times 10^3 + 12 \times 10^3 = 15 \times 10^3$ cm $= 150$ m.

From straight line equation,

$v = u + at \implies 0 = 3 \times 10^3 + a \times 2$

$$a = \frac{-3\times10^3}{2} = -1.5\times10^3 \text{ cm/s}^2 = -15 \text{ m/s}^2.$$

Illustration 5

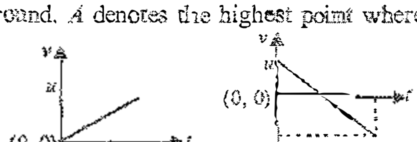
Indicate the velocity (v) - time (t) graphs for a body:

(i) falling under gravity.

(ii) thrown vertically upwards till it falls back to the ground after reaching the highest point.

Soln. : (i) Velocity increases from zero to say u

(ii) Velocity decreases from u to zero at the highest point. Then it falls under gravity to acquire the velocity u when it hits the ground. A denotes the highest point where $v = 0$.



(i) (ii)

Illustration 6

Motion of a particle is defined by $x = (3 - 4t + 5t^2)$ m. Graphically represent the variation of velocity $\left(\frac{dx}{dt}\right)$ with time.

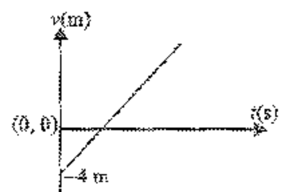
Soln. : $x = 3 - 4t + 5t^2$

$\therefore \quad \frac{dx}{dt} = -4 + 10t$

or velocity, $v = -4 + 10t$

At $t = 0$, $v = -4$ m.

Also $v \propto t$. It is a straight line graph.



Equation of Motion for a Uniformly Accelerated Motion

(i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 - u^2 = 2as$ (iv) $s_n = u + \frac{a}{2}(2n-1)$

where u is initial velocity, v is final velocity, a is uniform acceleration, s is distance travelled in time t, s_n is distance covered in n^{th} second. These equations are not valid if the acceleration is non-uniform.

Equation of Motion for a Freely Falling Body Under Gravity

(i) $v = u + gt$ (ii) $h = ut + \frac{1}{2}gt^2$

(iii) $v^2 = u^2 + 2gh$ (iv) $h_n = u + \frac{1}{2}g(2n-1)$

Illustration 7

From the top of a tower a stone is thrown up which reaches the ground in time t_1. A second stone thrown down with the same speed reaches the ground in time t_2. A third stone released from rest from the same location reaches the ground in time t_3. Show that $t_3^2 = t_1 t_2$.

Soln. : Let h = height of tower

u = speed of projection of stone

$$\therefore \quad h = -ut_1 + \frac{1}{2}gt_1^2 = ut_2 + \frac{1}{2}gt_2^2$$

But h is also $= 0 + \frac{1}{2}gt_3^2$

or $\quad ut_2 + ut_1 = \frac{1}{2}gt_1^2 - \frac{1}{2}gt_2^2$

or $\quad u(t_2 + t_1) = \frac{1}{2}g(t_1 + t_2)(t_1 - t_2)$

or $\quad u = \frac{1}{2}g(t_1 - t_2)$ ...(i)

$$\therefore \quad -\frac{1}{2}g(t_1 - t_2)t_1 + \frac{1}{2}gt_1^2 = \frac{1}{2}gt_3^2$$

or $-t_1^2 + t_1t_2 + t_1^2 = t_3^2$ or $t_1t_2 = t_3^2$

Illustration 8

A ball is dropped on to the floor from a height of 10 m. It rebounds to a height of 2.5 m. If the ball is in contact with the floor for 0.01 sec, what is the average acceleration during contact?

Soln. : Under free fall, velocity, $v_1 = \sqrt{2gh}$

$v_1 = \sqrt{2\times10\times10} = \sqrt{200}$ m/s

Under rise, velocity, $v_2 = -\sqrt{2\times10\times2.5} = -\sqrt{50}$ m/s

$\therefore$ Change in velocity $= (v_1 - v_2) = \sqrt{200} - (-\sqrt{50})$

$= \sqrt{200} + \sqrt{50} = 21$ m/s

$$\therefore \quad \text{Acceleration} = \frac{v_1 - v_2}{\text{time}} = \frac{21}{0.01} = 2100 \text{ m/s}^2$$

$\therefore$ Acceleration $= 2100$ m/s^2.

Illustration 9

A body is freely dropped from a height h above the ground.

(a) Find the ratio of distances fallen in first one second, first two seconds, first three seconds, etc.

(b) Find the ratio of distances fallen in 1st second, in 2nd second, in 3rd second etc.

Soln. :

(a) From equation $h = \frac{1}{2}gt^2 \quad \left[\because h = ut + \frac{1}{2}gt^2 \text{ and } u = 0\right]$

$$h_1 : h_2 : h_3 \ldots\ldots = \frac{1}{2}g(1)^2 : \frac{1}{2}g(2)^2 : \frac{1}{2}g(3)^2 : \ldots\ldots$$

$= 1^2 : 2^2 : 3^2 : \ldots\ldots$

$= 1 : 4 : 9 : \ldots\ldots$

(b) Now from the formula of distance travelled in n^{th} second

$s_n = u + \frac{1}{2}a(2n-1)$ where $u = 0, a = g$

$\therefore \quad s_n = \frac{1}{2}g(2n-1)$