or, $s_1 : s_2 : s_3 .... = \frac{1}{2}g(2\times1-1) : \frac{1}{2}g(2\times2-1) : \frac{1}{2}g\ (2\times3-1) :....$

$= 1 : 3 : 5 ....$

VECTORS

- The physical quantities which have both magnitude as well as direction are known as vector quantities or vectors. *e.g.*, force, velocity etc.
- **Representation of a vector :** A vector is represented by a straight line with arrow head on it. The length of the line represents the magnitude of a vector and arrow head tells the direction of vector. In writing, a vector is represent by a single letter with a arrow head on it, *e.g.*, the force is a vector quantity, it is represented by $\vec{F}$.
- **Vectors are of two types**

(i) Polar vectors : A polar vector involves a displacement or virtual displacement *e.g.*, velocity, force, etc.

(ii) Axial vectors or Pseudo vectors: A pseudo vector or axial vector involves the orientation of an axis in space *e.g.*, angular velocity, torque, angular momentum, etc.

- **Equal vectors :** Two vectors are said to be equal if they have same magnitude and direction regardless of their initial positions.
- **Negative vector :** It is a vector having same magnitude but direction opposite to that of a given vector.
- **Null vector :** It is a vector whose magnitude is zero, but its direction is not defined.
- **Properties of a null vector**

(i) $\vec{A} + \vec{0} = \vec{A}$

(ii) $\lambda\vec{0} = \vec{0}$ where λ is a scalar.

(iii) $\vec{0}\vec{A} = \vec{0}$

- **Unit vector :** A vector having magnitude equal to unity. To find the unit vector in the direction $\vec{A}$, we divide the given vector by its magnitude.

e.g. $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

- **Coinitial vectors :** The vectors are said to be co-initial, if their initial point is common.
- **Collinear vectors :** These are those vectors which are having equal or unequal magnitudes and are acting along the parallel straight lines.
- **Coplanar vectors :** These are those vectors which are acting in the same plane.
- **Laws of vector algebra :** If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are vectors, and m and n are scalars, then

(i) Commutative Law for addition

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$

(ii) Associative Law for addition

$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

(iii) Associative Law for multiplication

$m(n\vec{A}) = (mn)\vec{A} = n(m\vec{A})$

(iv) Distributive Law

$(m+n)\vec{A} = m\vec{A} + n\vec{A}$

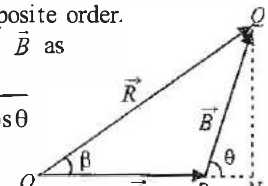
$m(\vec{A} + \vec{B}) = m\vec{A} + m\vec{B}$

- **Triangle law of vector addition:** It states that if two vectors can be represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant is represented completely (both in magnitude and direction) by the third side of the triangle taken in the opposite order.

$\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$ as shown in figure,

then $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$

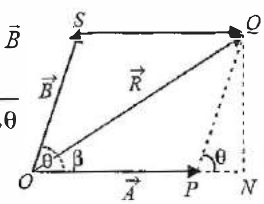


- **Parallelogram law of vector addition :** It states that if two vectors acting simultaneously at a point can be represented both in magnitude and direction by the two adjacent sides of a parallelogram, then the resultant is represented completely (both in magnitude and direction) by the diagonal of the parallelogram passing through that point.

$\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$ as shown in figure,

then $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$



- **Polygon law of vector addition :** It states that if a number of vectors can be represented both in magnitude and direction by the sides of a polygon taken in the same order, then their resultant is represented (both in magnitude and direction) by the closing side of the polygon taken in the opposite order.
- **Rectangular component of a vector in a plane :** When a vector is splitted into two component vectors at right angles to each other, the component vectors are called rectangular components of a vector. If $\vec{A}$ makes an angle θ with x-axis, and A_x and A_y be the rectangular components of $\vec{A}$ along x-axis and y-axis respectively, then

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

Here $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

$\therefore\ A_x^2 + A_y^2 = A^2(\cos^2\theta + \sin^2\theta)$

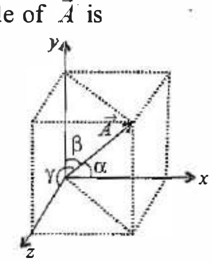
or $A = (A_x^2 + A_y^2)^{1/2}$ and $\tan\theta = A_y/A_x$.

- **Resolution of a vector in a space :** Let α, β, and γ are the angles between $\vec{A}$ and the x, y and z-axis, respectively as shown in the figure, then

$A_x = A\cos\alpha$, $A_y = A\cos\beta$, $A_z = A\cos\gamma$

In general, we have $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

The magnitude of $\vec{A}$ is



$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Dot or Scalar Product

The dot or scalar product of two vectors $\vec{A}$ and $\vec{B}$ denoted by $\vec{A}\cdot\vec{B}$ (read $\vec{A}$ dot $\vec{B}$) is defined as the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and the cosine of the angle between them. In symbols, $\vec{A}\cdot\vec{B} = AB\cos\theta,\ 0 \le \theta \le \pi$

Note $\vec{A}\cdot\vec{B}$ is a scalar and not a vector.

- **Properties of dot product**

(i) $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

(ii) $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

(iii) $m(\vec{A}\cdot\vec{B}) = (m\vec{A})\cdot\vec{B} = \vec{A}\cdot(m\vec{B}) = (\vec{A}\cdot\vec{B})m,$ where m is a scalar

(iv) $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1, \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

(v) If $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ and $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

$$\vec{A}\cdot\vec{B} = A_1B_1 + A_2B_2 + A_3B_3$$
$$\vec{A}\cdot\vec{A} = A^2 = A_1^2 + A_2^2 + A_3^2$$
$$\vec{B}\cdot\vec{B} = B^2 = B_1^2 + B_2^2 + B_3^2$$

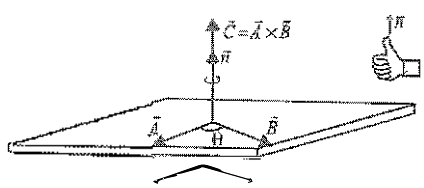
(vi) If $\vec{A}\cdot\vec{B} = 0$ and $\vec{A}$ and $\vec{B}$ are not null vectors, then $\vec{A}$ and $\vec{B}$ are perpendicular.

Cross or Vector Product

The cross or vector product of $\vec{A}$ and $\vec{B}$ is a vector $\vec{C} = \vec{A}\times\vec{B}$ (read $\vec{A}$ cross $\vec{B}$). The magnitude of $\vec{A}\times\vec{B}$ is defined as the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and $\vec{B}$ the sine of the angle between them. The direction of the vector $\vec{C} = \vec{A}\times\vec{B}$ is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ and such that $\vec{A}$, $\vec{B}$ and $\vec{C}$ form a right handed system. In symbols,

$$\vec{A}\times\vec{B} = AB\sin\theta\hat{n},\ 0 \le \theta \le \pi$$

where $\hat{n}$ is a unit vector indicating the direction of $\vec{A}\times\vec{B}$



Examples of vector product :

(i) Torque $(\vec{\tau}) = \vec{r}\times\vec{F}$

(ii) Angular momentum $(\vec{L}) = \vec{r}\times\vec{p}$

(iii) Velocity $(\vec{v}) = \vec{\omega}\times\vec{r}$

(iv) Acceleration $(\vec{a}) = \vec{\alpha}\times\vec{r}$

Here $\vec{r}$ is position vector or radius vector and $\vec{F}, \vec{p}, \vec{\omega}$ and $\vec{\alpha}$ are force, linear momentum, angular velocity and angular acceleration respectively.

- **Properties of vector product**

(i) $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

(ii) $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

(iii) $m(\vec{A}\times\vec{B}) = (m\vec{A})\times\vec{B} = \vec{A}\times(m\vec{B}) = (\vec{A}\times\vec{B})m,$ where m is a scalar.

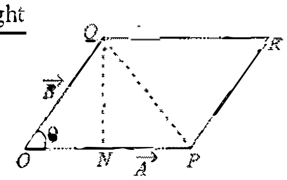
(iv) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0,\ \hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$

(v) If $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ and $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$, then

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

(vi) $\vec{A}\times\vec{B}$ = the area of a parallelogram with sides $\vec{A}$ and $\vec{B}$.

(vii) If $\vec{A}\times\vec{B} = 0$ and $\vec{A}$ and $\vec{B}$ are not null vectors, then $\vec{A}$ and $\vec{B}$ are parallel.



$$\text{Area of } \Delta POQ = \frac{\text{base}\times\text{height}}{2} = \frac{(OP)(NQ)}{2} = \frac{A\times B\sin\theta}{2} = \frac{1}{2}|\vec{A}\times\vec{B}|$$

$\therefore$ Area of parallelogram $OPRQ$ = 2[area of ΔOPQ] $= |\vec{A}\times\vec{B}|$

Illustration 10

The diagonals of a parallelogram are vectors $\vec{A}$ and $\vec{B}$. If $\vec{A} = 5\hat{i} - 4\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j} - \hat{k}$. Calculate the magnitude of area of this parallelogram.

Soln. : When $\vec{A}$ and $\vec{B}$ are the diagonals of a parallelogram, then its

$$\text{Area} = \frac{1}{2}|\vec{A}\times\vec{B}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -4 & 3 \\ 3 & -2 & -1 \end{vmatrix}$$

$$= \frac{1}{2}|\hat{i}(4+6) - \hat{j}(-5-9) + \hat{k}(-10+12)|$$

$$= \frac{1}{2}|10\hat{i} + 14\hat{j} + 2\hat{k}| = \frac{1}{2}\sqrt{(10)^2 + (14)^2 + (2)^2}$$

$$= \frac{1}{2}\sqrt{300} = \frac{10}{2}\sqrt{3} = 5\sqrt{3} \text{ unit}$$

Illustration 11

If $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$, then find the angle between $\vec{A}$ and $\vec{B}$.

Soln. : $\because$ $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

$\therefore$ $\sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

or $A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

or $4AB\cos\theta = 0$ or $\cos\theta = 0$

$\therefore$ $\theta = 90^\circ$

Illustration 12

The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled then the new resultant vector is perpendicular to $\vec{P}$. Then R is equal to

(a) $\left(\dfrac{P^2 - Q^2}{2PQ}\right)$ (b) Q (c) $\dfrac{P}{Q}$ (d) $\dfrac{P+Q}{P-Q}$