CIRCULAR MOTION

- Circular motion is a two dimensional motion.
- **Angular displacement :** The angular displacement of an object moving around a circular path is defined as the angle traced out by the radius vector at the centre of the circular path in a given time. It is denoted by symbol θ.
- **Angular velocity :** Angular velocity of an object in circular motion is defined as the rate of change of its angular displacement with time. It is generally denoted by symbol ω (omega) and it is given by

$$\omega = \frac{d\theta}{dt}$$

- SI unit of angular velocity is rad s^{-1} and its dimensional formula is $[M^0L^0T^{-1}]$.
- **Angular acceleration :** Angular acceleration of an object in circular motion is defined as the rate of change of its angular velocity with time. It is generally denoted by symbol α and it is given by

$$\alpha = \frac{d\omega}{dt}$$

- SI unit of angular acceleration is rad s^{-2} and its dimensional formula is $[M^0L^0T^{-2}]$.
- **Uniform circular motion :** When an object is moving on a circular path with a constant speed then the motion of an object is said to be a uniform circular motion.
- **Time period :** In circular motion, the time period is defined as the time taken by an object to complete one revolution on its circular path. It is generally denoted by symbol T and is expressed in second.
- **Frequency :** In circular motion, the frequency is defined as the number of revolutions completed by an object on its circular path in a unit time. It is generally denoted by υ. Its unit is s^{-1} or hertz (Hz).
- **Relation between time and frequency**

$$T = \frac{1}{\upsilon} \quad \text{or} \quad \upsilon = \frac{1}{T}$$

- **Relation between angular velocity, frequency and time period**

$$\omega = \frac{2\pi}{T} = 2\pi\upsilon$$

- **Centripetal acceleration :** Acceleration acting on an object undergoing uniform circular motion is known as centripetal acceleration. It is given by

$$\text{Centripetal acceleration, } a_c = \frac{v^2}{r} = \omega^2 r$$

where r is the radius of the circle.
- It always acts on an object along the radius towards the centre of the circular path.
- Centripetal acceleration is not constant vector.

Tangential Acceleration and Radial Acceleration

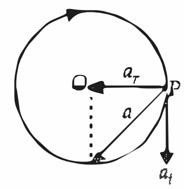
- Acceleration of an object in circular motion (or motion along any curved path) has two components :
  - **Tangential acceleration :** The tangential acceleration arises from the change in the speed of an object and has a magnitude given by

$$a_t = \frac{d|\vec{v}|}{dt}$$

  - **Radial acceleration :** The radial acceleration is due to the change in direction of the velocity and is given by

$$a_r = \frac{v^2}{r} = \omega^2 r$$

As a_t and a_r are perpendicular to each other as shown in figure.



$\therefore$ Net acceleration, $|\vec{a}| = \sqrt{a_t^2 + a_r^2}$

In uniform circular motion, $a_t = 0$ but $a_r \neq 0$

Illustration 24

An astronaut is rotating in a rotor of radius 4 m. If he can withstand upto acceleration of 10 g, then what is the maximum number of permissible revolutions? ($g = 10$ m s^{-2})

Soln. : In case of uniform circular motion

$$a_r = \frac{v^2}{r} = \omega^2 r \quad [\text{as } v = r\omega]$$

or $a_r = (2\pi f)^2 r$ [as $\omega = 2\pi f$]

or $f = \frac{1}{2\pi}\sqrt{\frac{a_r}{r}}$

Hence $a_r < 10$ g, so

$$f < \frac{1}{2\pi}\sqrt{\frac{10\times10}{4}} \quad i.e.\ f_{max} = \left[\frac{5}{2\pi}\right]\text{rev/sec}$$

Illustration 25

A motor car is travelling at 30 m s^{-1} on a circular road of radius 500 m. It is increasing its speed at the rate of 2 m s^{-2}, what is its acceleration?

Soln. : In this problem,

$$a_t = \frac{dv}{dt} = 2 \text{ m s}^{-2}$$

$$a_r = \frac{v^2}{r} = \frac{30\times30}{500} = 1.8 \text{ m s}^{-2}$$

so, $a = \sqrt{a_r^2} = \sqrt{(1.8)^2 + (2)^2} = 2.7$ m s^{-2}