UNIT 3

Laws of Motion

INERTIA

- It is an inherent property of all the bodies by virtue of which they cannot change their state of rest or uniform motion along a straight line on their own. Quantitatively, inertia of a body is measured by its mass. Thus, heavier the body greater is its inertia.

Types of Inertia

Inertia of a body is of three types :

- Inertia of rest
- Inertia of motion
- Inertia of direction

- **Inertia of rest :** It is the inability of a body to change its state of rest by itself. *e.g.*, a person standing in a stationary bus falls backward when the bus suddenly starts moving.
- **Inertia of motion :** It is the inability of a body to change its state of uniform motion by itself. *e.g.*, a person getting down a moving bus or train falls forward.
- **Inertia of direction :** It is the inability of a body to change its direction of motion by itself. *e.g.*, when a car suddenly takes a turn, the person sitting inside is thrown in the outward direction.

NEWTON S LAWS OF MOTION

Newton s First Law of Motion

- According to Newton's first law, every body continues to be in its state of rest or of uniform motion in a straight line, unless it is compelled by some external force to act otherwise.
- **Newton's first law of motion can be expressed as :** If the net external force on a body is zero, its acceleration is zero. Acceleration can be non-zero only if there is a net external force on the body.
- Newton's first law is also known as law of inertia.
- **Momentum :** Momentum of a body $(\vec{p})$ is defined as the product of its mass (m) and velocity $(\vec{v})$.

  i.e., $\vec{p} = m\vec{v}$

  The direction of momentum is same as that of the velocity.
- Momentum is a vector quantity. Its SI unit is kg m s^{-1}. Its dimensional formula is [MLT^{-1}].

Newton s Second Law of Motion

- According to Newton's second law of motion, the rate of change of momentum is directly proportional to the applied force and takes place in the direction in which force acts.

  $\vec{F} = k\frac{d\vec{p}}{dt} = km\vec{a}$, as m is constant.

  where $\vec{F}$ is the net external force on the body and $\vec{a}$ its acceleration. In both SI and CGS systems constant of proportionality $k = 1$.
- Newton's second law of motion gives us a measure for force.
- Newton's second law is consistent with the first law ($\vec{F} = 0$ implies $\vec{a} = 0$).
- Newton's second law of motion is applicable to a particle, and also to a body or a system of particles provided $\vec{F}$ is the total external force on the system and $\vec{a}$ is the acceleration of system as a whole.
- **Units of force :** The units of force are of two types
  - Absolute unit
  - Gravitational unit
- **Absolute unit :** In SI system the absolute unit of force is newton. It is denoted by symbol N.

  1 N = 1 kg m s^{-2}

  In CGS system the absolute unit of force is dyne.

  1 dyne = 1 g cm s^{-2}
- Relationship between newton and dyne

  1 N = 10^5 dyne
- **Gravitational unit :** In SI system, the gravitational unit of force is kilogram weight (kg wt) or kilogram force (kg f).

  1 kg wt or 1 kg f = 9.8 N.

  In CGS system, the gravitational unit of force is gram weight (g wt) or gram force (g f).

  1 g wt or 1 g f = 980 dyne
- The gravitational unit of force is used to express the weight of a body. *e.g.*, weight of a body of a mass 2 kg is 2 kg f or 2 kg wt.
- Force is a vector quantity. Its dimensional formula is [MLT^{-2}].
- The straight line along which a force is directed is called line of action of force.

Illustration 1

A force $\vec{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})$ N produces acceleration of 1 m s^{-2} in a body. Calculate the mass of the body.

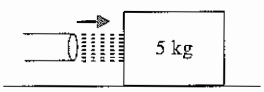
Soln. : $\because\ a = \frac{|\vec{F}|}{m}$

$$\therefore\ m = \frac{|\vec{F}|}{a} = \frac{\sqrt{(6)^2 + (-8)^2 + (10)^2}}{1} = 10\sqrt{2}\text{ kg}$$

Illustration 2

A block of 5 kg is resting on a frictionless plane. It is struck by a jet releasing water at the rate of 3 kg/s at a speed of 4 m s^{-1}. Calculate the initial acceleration of the block.

Soln. : Force exerting on block

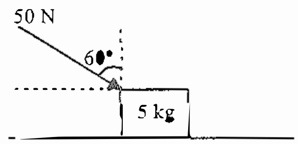


$$F = v\frac{dm}{dt} = 4 \times 3 = 12\text{ N}$$

So acceleration of the block, $a = \frac{F}{m} = \frac{12}{5} = 2.4\text{ m s}^{-2}$

Illustration 3

A force of 50 N acts in the direction as shown in figure. The block of mass 5 kg, resting on a smooth horizontal surface. Find the acceleration of the block.



Soln. : Horizontal component of force

$$= F\sin\theta = 50\sin 60° = \frac{50\sqrt{3}}{2}$$

acceleration of the block, $a = \frac{F}{m} = \frac{50\sqrt{3}}{2} \times \frac{1}{5} = 5\sqrt{3}\text{ m s}^{-2}$

Newton s Third Law of Motion

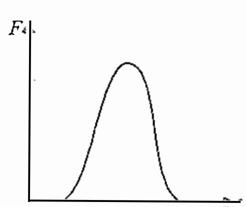
- According to Newton's third law of motion, to every action, there is anequal and opposite reaction.
- **Newton's third law of motion can be stated as :** Forces in nature always occur between pairs of bodies. Force on a body A by body B is equal and opposite to the force on the body B by body A.
- Action and reaction forces are simultaneous forces. There is no cause-effect relation between action and reaction. Any of the two mutual forces can be called action and the other reaction. Action and reaction act on different bodies and so they cannot be cancelled out.

IMPULSE

Impulse is defined as the product of the force and time for which that force acts on the body which equals change in momentum.

i.e., Impulse = force × time duration
= change in momentum.

- Suppose a force $\vec{F}$ acts for a short time dt then impulse $= \vec{F}dt$
- For a finite interval of time from t_1 to t_2 then the impulse $= \int_{t_1}^{t_2} \vec{F}dt$



Impulse = area under

- If constant force acts for an interval Δt then impulse $= \vec{F}\Delta t$

  $\therefore\quad \vec{F}\Delta t = \Delta\vec{P}$
- Impulse is a vector quantity. Its direction is same as that of the force.
- The SI unit of impulse is N s and its CGS unit is dyne s. Its dimensional formula is same as that of momentum *i.e.*, [MLT^{-1}].
- Impulse is equal to the area under the force-time graph.
- **Impulsive force :** A large force acting for a short time to produce a finite change in momentum is known as impulsive force.

LAW OF CONSERVATION OF LINEAR MOMENTUM

- It states that when no external force acts on a system, total linear momentum of the system remain constant.
- The law of conservation of momentum is one of the most fundamental law of mechanics.
- Recoil velocity of a gun : $V = -mv/M$

  where m = mass of the bullet, M = mass of the gun, v = velocity of the bullet.

  negative sign shows that the gun recoils in a direction opposite to that of the bullet.
- Rocket propulsion is an example of variable mass.
- Newton's third law of motion and the principle of conservation of linear momentum form the basis of rocket propulsion.
- Thrust on the rocket, $F = -u\frac{dm}{dt}$

  Negative sign shows that thrust on the rocket is in a direction opposite to the direction of escaped gases.
- Velocity of rocket, $v = v_0 + u\ln\left(\frac{m_0}{m}\right)$

  where u = velocity of the ejected gases relative to the rocket, v_0 = initial velocity of the rocket, m_0 = initial mass of the rocket, dm/dt = rate of combustion of fuel at the instant t, m = mass of the rocket at time t.

Illustration 4

A man weighing 80 kg is standing on a trolley weighing 320 kg. The trolley has frictionless horizontal rails available for its motion. If the man starts walking on the trolley along the rails at speed 1 m/s, then calculate his displacement relative to ground in 4 sec.

Soln. : Momentum is conserved. System consists of man and trolley.

Momentum of system = Momentum of the man

$(320 + 80)\,v = 80 \times 1 \Rightarrow v = 0.2\text{ m s}^{-1}$

Displacement of man = 4 × 1 = 4 m

Displacement of system = (0.2) × 4 = 0.8 m

∴ Displacement of man relative to ground = (4 – 0.8) = 3.2 m

Illustration 5

A cricket ball of mass 0.2 kg moves with a velocity of 20 m/s and is brought to rest by a player in 0.1 s. Calculate the impulse of the ball and average force applied by the player.

Soln. : Impulse $= P_2 - P_1 = mv - mu = 0 - 0.2 \times 20 = -4$ N s

Average force, $F = \frac{P_2 - P_1}{t} = \frac{-4}{0.1} = -40$ N