Illustration 6

A hammer of mass 1 kg moving with a speed of 6 m s^{-1} strikes a wall and comes to rest in 0.1 sec. Calculate

(a) Impulse of the force

(b) Average retarding force that stops the hammer

(c) Average retardation of the hammer

Soln. : $m = 1$ kg, $u = 6$ m/sec, $v = 0$, $t = 0.1$ s

(a) Impulse $= Ft = m(v - u) = 1(0 - 6) = -6$ N s

(b) Average retarding force that stops the hammer

$$F = \frac{6}{0.1} = 60\text{ N}$$

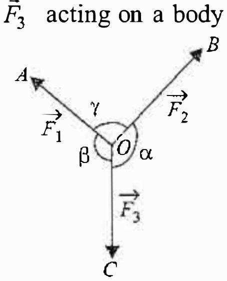
(c) Average retardation, $a = \frac{F}{m} = \frac{60}{1} = 60\text{ m/sec}^2$

EQUILIBRIUM OF CONCURRENT FORCES

- **Concurrent forces :** Forces acting together on a body at the same point are called concurrent forces.
- A number of concurrent forces acting on a body are said to be in equilibrium, if the resultant of these forces is zero or if the concurrent forces can be represented completely by the sides of a closed polygon taken in the same order.

  Mathematically : $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_n = 0$
- **Lami's theorem :** According to Lami's theorem, when three concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ acting on a body are in equilibrium, then

  $$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



  where

  α = angle between $\vec{F}_2$ and $\vec{F}_3$

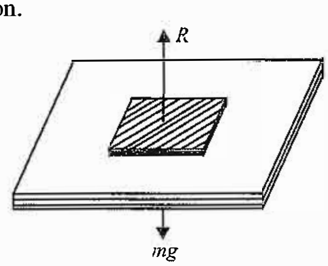
  β = angle between $\vec{F}_3$ and $\vec{F}_1$

  γ = angle between $\vec{F}_1$ and $\vec{F}_2$

Types of Forces

- **Weight :** It is the force which the earth attracts any body to itself. It is given by $W = mg$

  where m is the mass of the body and g is the acceleration due to gravity.
- **Reaction :** When a body is pressed against a rigid surface, the body experiences a force which is perpendicular to the surfaces in contact. This force is called normal force or reaction.



- **Spring force :** The spring force is given by $F = -kx$ where x represents the enlongation or compression of the spring and k is the force constant of the spring.

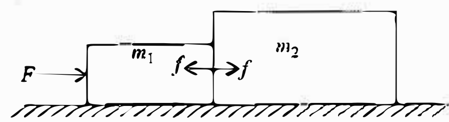
APPARENT WEIGHT OF A MAN IN A LIFT

- When the lift is at rest or moving with constant velocity, the apparent weight = mg. Thus, apparent weight = true weight.
- When the lift is accelerating upwards with acceleration a, then apparent weight = $m(g + a)$. Thus, apparent weight is more than the true weight.
- When the lift is accelerating downwards with acceleration a, then apparent weight = $m(g - a)$. Thus, apparent weight is less than the true weight of man.

- If the cable supporting the lift breaks, the lift falls freely with $a = g$, then apparent weight = $m(g - g) = 0$.
- When a person of mass m climbs up a rope with acceleration a, the tension in the rope is $T = m(g + a)$.
- When the person climbs down the rope with acceleration a, the tension in the rope is $T = m(g - a)$.
- When the person climbs up or down with uniform speed, the tension in the rope is $T = mg$

MOTION OF BODIES IN CONTACT

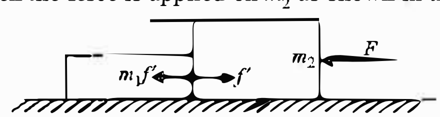
- Two bodies of masses m_1 and m_2 respectively are placed in contact with each other on a smooth horizontal surface. Let a force F be applied on m_1 as shown in the figure.



  - Common acceleration of the system,

    $$a = \frac{F}{m_1 + m_2}$$
  - Contact force between m_1 and m_2,

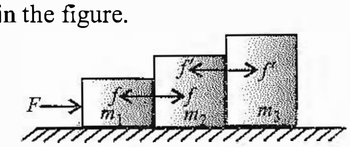
    $$f = \frac{m_2 F}{m_1 + m_2}$$
- When the force is applied on m_2 as shown in the figure.



  - Common acceleration of the system ,

    $$a = \frac{F}{m_1 + m_2}$$
  - Contact force between m_1 and m_2,

    $$f' = \frac{m_1 F}{m_1 + m_2}$$
- When three bodies of masses m_1, m_2 and m_3 respectively are placed in contact with each other on a smooth horizontal surface. Let a force F be applied on m_1 as shown in the figure.



  - Common acceleration of the system,

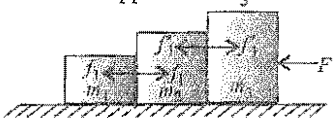
    $$a = \frac{F}{m_1 + m_2 + m_3}$$
  - Contact force between m_1 and m_2,

    $$f = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$$

- Contact force between m_2 and m_3,

$$f' = \frac{m_3 F}{m_1 + m_2 + m_3}$$

- When the force F is applied on m_3 as shown in the figure.



  - Common acceleration of the system,

$$a = \frac{F}{m_1 + m_2 + m_3}$$

  - Contact force between m_1 and m_2,

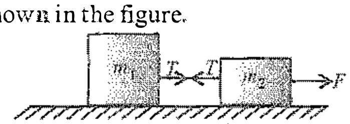
$$f_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$$

  - Contact force between m_2 and m_3,

$$f_1' = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$$

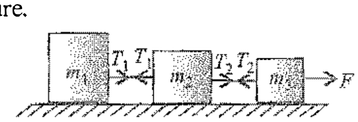
MOTION OF CONNECTED BODIES

- When two bodies of masses m_1 and m_2 respectively are connected by a uniform massless string and placed on a smooth horizontal surface. Let a force F be applied on m_2 as shown in the figure.



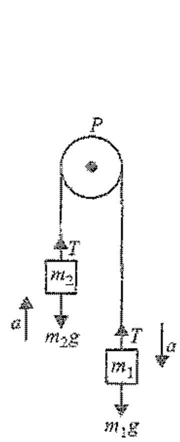
  - Common acceleration of the system, $a = \dfrac{F}{m_1 + m_2}$
  - Tension, $T = \dfrac{m_1 F}{m_1 + m_2}$

- When three bodies of masses m_1, m_2 and m_3 respectively are connected by the strings and placed on a smooth horizontal surface. Let a force F is applied on m_3 as shown in the figure.



  - Acceleration of the system, $a = \dfrac{F}{m_1 + m_2 + m_3}$
  - Tension, $T_1 = \dfrac{m_1 F}{m_1 + m_2 + m_3}$
  - Tension, $T_2 = \dfrac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

- When two bodies of masses m_1 and m_2 $(m_1 > m_2)$ connected by an inextensible string passing over a light frictionless pulley as shown in the figure



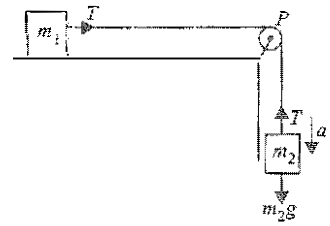
  - Acceleration of the system,

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

  - Tension in the string,

$$T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

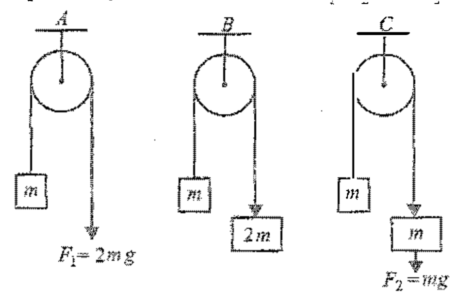
- When two bodies of masses m_1 and m_2 are attached to the two ends of inextensible string passing over a frictionless pulley attached to an edge of a horizontal table as shown in the figure.



- Acceleration of the system, $a = \dfrac{m_2 g}{m_1 + m_2}$
- Tension in the string, $T = \dfrac{m_1 m_2 g}{m_1 + m_2}$

Illustration 7

In figure A, B and C, each block have acceleration a_1, a_2 and a_3 respectively. F_1 and F_2 are external forces of magnitude $2mg$ and mg respectively. Find the value of a_1, a_2 and a_3.

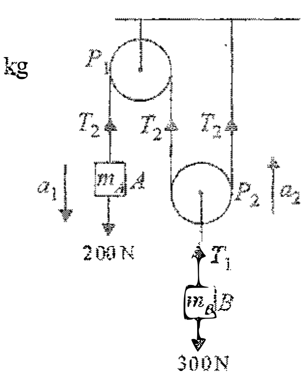


Soln. : $a_1 = \dfrac{2mg - mg}{m} = g$, $a_2 = \dfrac{2m - m}{2m + m}g = \dfrac{g}{3}$

$$a_3 = \frac{mg + mg - mg}{2m} = \frac{g}{2}, \text{ clearly } a_1 > a_3 > a_2$$

Illustration 8

A pulley system is shown in figure, pulley P_1 is fixed to a rigid support and pulley P_2 is capable of moving freely upward or downward. The pulleys and strings are ideal. Weights of two masses A and B are 200 N and 300 N respectively. Find the tensions T_1 and T_2 and also the acceleration of A and B $(g = 10 \text{ m s}^{-2})$.



Soln. : $m_A = \dfrac{200 \text{ N}}{g} = \dfrac{200}{10} = 20 \text{ kg}$

and $m_B = \dfrac{300 \text{ N}}{g} = 30 \text{ kg}$

Because the strings are ideal the tension along a string remains the same. The forces acting on the pulley P_2 are $2T_2$ upwards and T_1 downwards.

$$T_1 = 2T_2$$

The force lifting B up is $2T_2$ while one pulling A upwards is only T_2.

Hence, mass B rises up while A moves downwards.

Let the acceleration of $A = \frac{a_1}{1}$, downwards and acceleration of $B = a_2$ upwards.

Since for a displacement x of A downwards, B suffers a displacement only $\frac{x}{2}$.

Hence, $a_1 = 2a_2$

Equation of motion for A

$$200 - T_2 = 20a_1 \quad ...(i)$$

Equation of motion for B

$$T_1 - 300 = 30a_2 \Rightarrow 2T_2 - 300 = 30 \times \frac{a_1}{2}$$

$$\Rightarrow 2T_2 - 300 = 15a_1 \quad ...(ii)$$

On solving (i) and (ii), $a_1 = \frac{100}{55} = 1.818 \text{ m s}^{-2}$

$$\therefore \quad a_2 = \frac{a_1}{2} = 0.909 \text{ m s}^{-2}$$

and $T_1 = 30a_2 + 300 = 27.27 + 300 = 327.27$ N

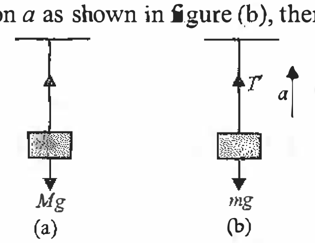
$$T_2 = \frac{T_1}{2} = 163.64 \text{ N}$$

Illustration 9

A rope of negligible mass can support a load of M kg. Prove that the mass of the greatest load which can be raised is equal to $\frac{M}{1 + \frac{2h}{gt^2}}$ kg, where g is the acceleration due to gravity and h is the height through which the said load rises from rest with uniform acceleration in time t.

Soln. : Since the rope can support a load of M kg, the maximum tension the rope can withstand is given by $T = Mg$ (figure a).

Now, if a mass m is raised by the rope with a uniform acceleration a as shown in figure (b), then the net



Upward force on the mass $m = (T' - mg)$ where T' is the tension in the string.

Then, from Newton's second law of motion,

$$T' - mg = ma \quad \text{or} \quad T' = m(g + a)$$

For the greatest value of m,

$$T' = T \quad \therefore m(g + a) = Mg$$

$$\therefore \quad m = \frac{Mg}{g + a} = \frac{M}{1 + a/g} \quad ...(i)$$

Now, from the equation of motion, $s = ut + \frac{1}{2} a t^2$

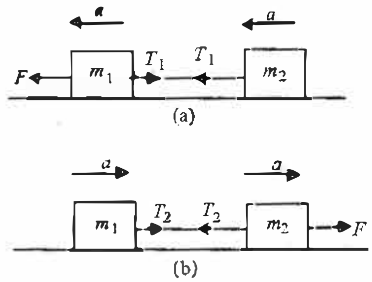
We get, $h = 0 + \frac{1}{2} a t^2$ or $a = \frac{2h}{t^2}$

From equation (i), $m = \frac{M}{1 + \frac{2h}{gt^2}}$ kg.

Illustration 10

A horizontal force of 500 N pulls two masses 10 kg and 20 kg lying on a frictionless table and connected by a light string. What is the tension in the string? Does the answer depend on which mass end the pull is applied?

Soln. :



Here, $m_1 = 10$ kg, $m_2 = 20$ kg and $F = 500$ N

Case (a) : When the force F is applied on mass m_1: Let T_1 be the tension in the connecting string. The free body diagram with all the forces in the horizontal directions of the masses m_1 and m_2 are shown in figure (a).

Now, applying Newton's second law of motion to masses m_1 and m_2 we get.

$$F - T_1 = m_1 a \quad ...(i)$$

and $T_1 = m_2 a \quad ...(ii)$

Eliminating a from eqns (i) and (ii) we get,

$$\frac{F - T_1}{m_1} = \frac{T_1}{m_2} \text{ or } T_1 = \frac{m_2 F}{m_1 + m_2} = \frac{20 \times 500}{10 + 20} = 333.3 \text{ N}$$

Case (b) : When the force is applied on mass m_2. With reference to figure (b), we can write,

$$T_2 = m_1 a \quad ...(iii)$$

and $F - T_2 = m_2 a \quad ...(iv)$

Again eliminating a from eqns (iii) and (iv) we get,

$$T_2 = \frac{m_1 F}{m_1 + m_2} = \frac{10 \times 500}{10 + 20} = 166.7 \text{ N}.$$

So, the tensions in both the cases are different, i.e., it depends on which mass the force is applied.

It may be noted that the acceleration is same in both the cases.

Illustration 11

A uniform rope of length L resting on a frictionless horizontal surface is pulled at one end by a force F. What is the tension in the rope at a distance l from the end where the force is applied?

Soln. : Let ABC be the rope and the force F be applied at the end A (see figure). We have to find out the tension at point B of the rope. If m is the total mass of the rope, the mass/unit length $= \frac{m}{L}$