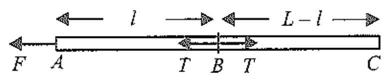
Acceleration of the rope $= \frac{F}{m}$.

Considering the motion of the portion BC of the rope. We have,

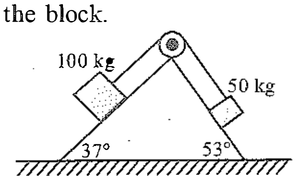


Net force on BC = mass of $BC \times$ acceleration

$$\therefore \text{Tension, } T = \frac{m}{L} \times (L-l) \times \frac{F}{m} = \frac{F}{L}(L-l).$$

Illustration 12

Two blocks are attached to the two ends of a string passing over a smooth pulley as shown in the figure. Find the acceleration of the block.



Soln. : $100g \sin 37° - T = 100a$

$T - 50g \sin 53° = 50a$

$\therefore\ 100g \sin 37° - 50g \sin 53° = 150 \times a$

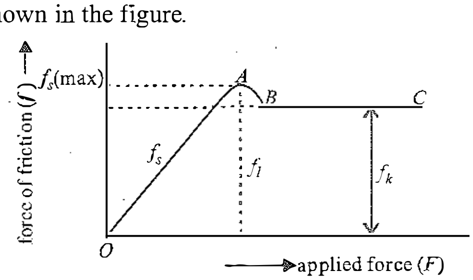
or $(100 \times 9.8 \times 0.6) - (50 \times 9.8 \times 0.8) = 150 \times a$

$\therefore\ a = 1.31 \text{ m/s}^2$.

FRICTION

- It is an opposing force which comes into play when one body actually moves (slides or rolls) or even tries to move over the surface of another body.

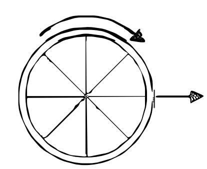
  Frictional forces arise due to interlocking of irregularities present on the two surfaces in contact. According to modern view, the frictional force arises due to strong atomic or molecular forces of attraction between the two surface at the points of actual contact.
- **Static friction :** It is an opposing force that comes into play when a body tends to move over the surface of another, but the actual motion has not yet started. It is a self adjusting force.
- **Limiting friction :** It is the maximum value of static friction. Limiting friction is the maximum opposing force that comes into play, when one body is just on the verge of moving over the surface of the other body.
- **Dynamic or kinetic friction :** It is the opposing force that acts between two surfaces in contact when one body is actually moving over the surface of another body.
- A graph between the force of friction and applied force is shown in the figure.



In the graph, curve OA represents the static friction (f_s) which is increasing with the applied force. At point A, the static friction is maximum which is known as the limiting friction (f_l). The part BC of the curve represents dynamic or kinetic friction (f_k) which is slightly lower than the static friction and remain constant.

- **Sliding friction :** It is an opposing force that comes into play when one body is actually sliding over the surface of the other body.
- **Rolling friction :** It is an opposing force that comes into play when one body is actually rolling over the surface of the other body.
- If a body rolls over the surface of another body, then both the rolling body and the surface on which it rolls are compressed by a small amount. The rolling body has to continuously climb a hill as shown in figure. The rolling body has to continuously detach itself from the surface on which it rolls. This is opposed by adhesive force between the two surfaces in contact and hence a force originates which retards the rolling motion. This retarding force is called the rolling friction. It is denoted by f_r.

$$f_r = \mu_r \times \frac{R}{r} \qquad \text{...(i)}$$



where μ_r is the coefficient of rolling friction, R is the normal reaction and r is the radius of the rolling body.

Methods of Reducing Friction

- By polishing the surface (But extra polishing increase friction).
- By lubrication.
- By proper selection of material.
- By avoiding moisture.
- By use of alloys.
- By streamlining the shape.
- By using ball bearings or roller bearings.

Laws of Friction

- The magnitude of the force of static friction between any two surfaces in contact can have the values

  $$f_s \le \mu_s R \qquad \text{... (i)}$$

  where the dimensionless constant μ_s is called the coefficient of static friction, R is the magnitude of normal reaction force. The equality in equation (i) holds when the surfaces are on the verge of slipping *i.e.*,

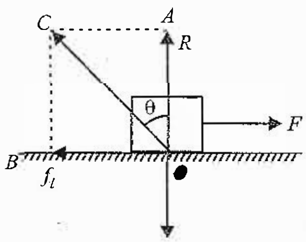
  $$f_s = (f_s)_{max} = (f_l) = \mu_s R.$$
- The magnitude of the force of kinetic friction acting between two surfaces is

  $$f_k = \mu_k R$$

  where μ_k is coefficient of kinetic friction.
- The value of μ_k and μ_s depend on the nature of the surfaces but μ_k is generally less than μ_s.
- The direction of the frictional force on a body is opposite to the actual motion (kinetic friction) or the impending motion (static friction) of the body relative to the surface with which it is in contact.

- The force of limiting friction is independent of the apparent area of contact, as long as normal reaction between the two bodies in contact remains the same.
- The force of limiting friction between any two bodies in contact depends on the nature of material of the surfaces in contact and their roughness or smoothness.

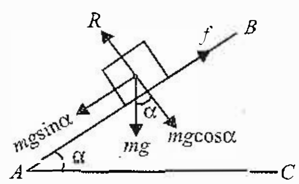
- **Angle of friction :** It is defined as the angle which the resultant of normal reaction (R) and the force of limiting friction (f_l) makes with the direction of normal reaction (R).



It is given as

$\tan\theta = \mu_s$ or $\theta = \tan^{-1}\mu_s$ ... (i)

- **Angle of repose :** Angle of repose is defined as the minimum angle of inclination of a plane with the horizontal such that a body placed on the plane just begins to slide down. It is represented by α.



It is given by $\tan\alpha = \mu_s$ or $\alpha = \tan^{-1}\mu_s$ ... (ii)

From the equations (i) and (ii), it follows that $\theta = \alpha$.

i.e., angle of repose = angle of friction.

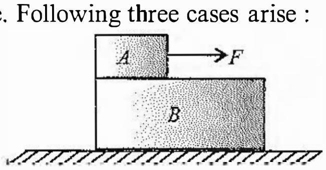
Though friction opposes motion, yet in certain cases, friction is also the cause of motion. For example,

- In moving, a person pushes the ground backwards (action) and the rough surface of ground reacts and exerts a forward force due to friction which causes the motion. Without friction, person would slip and shall not be able to move.
- In cycling, the rear wheel moves by the force communicated to it by pedalling while front wheel moves by itself. So, when pedalling a bicycle, the force exerted by rear wheel on ground makes force of friction act on it in the forward direction (like walking). Front wheel moves by itself as it experiences force of friction in backward direction (like rolling of a ball). However, if pedalling is stopped both wheels move by themselves and so experience force of friction in backward direction.

Acceleration of a body down a rough inclined plane $a = g(\sin\theta - \mu\cos\theta)$

where θ is the angle of inclination and μ is the coefficient of friction.

When a body A of mass m is resting on a body B of mass M and a force F is applied to the upper body as shown in the figure. Following three cases arise :



Frictionless

- When there is no friction

$$a_A = \frac{F}{m}, a_B = 0$$

- When there is friction between A and B only, the body A will not slide on B till

  $F < f_L$ *i.e.*, $F < \mu_s mg$

  Both A and B will move together with common acceleration

$$a_A = a_B = \frac{F}{M+m}$$

- When $F > f_L$, the two bodies will move in the direction of applied force, but with different accelerations.

  Force of dynamic friction, $f_k = \mu_k mg$

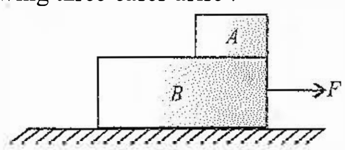
This causes the motion of B, $f_k = Ma_B$

$$\therefore a_B = \frac{f_k}{M} = \frac{\mu_k mg}{M}$$

For motion of A ; $F - f_k = ma_A$

$$\therefore a_A = \frac{(F - f_k)}{m} = \frac{(F - \mu_k mg)}{m}$$

- When a body A of mass m is resting on a body B of mass M and a force F is applied to the lower body as shown in figure. Following three cases arise :



Frictionless

- When there is no fiction

$$a_B = \frac{F}{M}$$

  $a_A = 0$,

  because there is no pulling force on A relative to B ; A will move backwards and so will fall from it after some time.

- When there is fiction between A and B only, and both are moving together, then

$$a = \frac{F}{m+M}$$

  The force on A will be

$$F' = ma = \frac{mF}{(m+M)}$$

  Note that both bodies will move together only when $F' < f_L$ ($f_L = \mu_s mg$)

  i.e. $\frac{mF}{m+M} < \mu_s mg$ or $F < \mu_s (m+M) g$

  Both bodies will move together with acceleration

$$a_A = a_B = \frac{F}{m+M} \text{ if } F < \mu_s (m+M) g$$

- If $F > \mu_s (m+M)g$, the bodies A and B will move with different accelerations ; such that

  $ma_A = \mu_k mg$

  $a_A = \mu_k g$