UNIT 4

Work, Energy and Power

WORK

- The measure of the work done by a force is the product of the force and the distance through which its point of application moves in the direction of the force.
- Work done by the force is defined as the dot product of force and displacement.

  i.e., $W = \vec{F} \cdot \vec{S} = FS\cos\theta$

  where θ is the angle between force and displacement.

Nature of Work

- Work done is positive, if the angle between $\vec{F}$ and $\vec{S}$ is acute (*i.e.* $\theta < 90°$) and work done is negative if angle between $\vec{F}$ and $\vec{S}$ is obtuse (*i.e.* $\theta > 90°$). For example when a body is pulled on a rough surface, the work done by the pulling force is positive, while work done by frictional force is negative.
- Work done is zero, if the angle between $\vec{F}$ and $\vec{S}$ is 90° (*i.e.* $\theta = 90°$). For example, when a coolie carrying some load on his head moves on a horizontal platform. Work done by the coolie is zero.
- Work is a scalar quantity. Its dimensional formula is $[ML^2T^{-2}]$.
- Unit of work are of two types:
  - Absolute unit
  - Gravitational unit
- **Absolute unit :** In SI system the absolute unit of work is joule. It is represented by J.

  $1\ J = 1\ N \times 1\ m.$

  In CGS system, the absolute unit of work is erg.

  $1\ erg = 1\ dyne \times 1\ cm$
- Relation between joule and erg

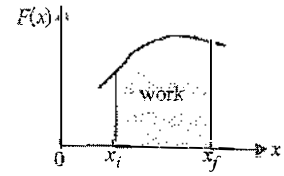
  $1\ joule = 10^7\ erg$
- **Gravitational unit :** In SI system the gravitational unit of work is kg m.

  $1\ kg\ m = 1\ kgf \times 1\ m = 9.8\ J.$

  In CGS system gravitational unit of work is g cm.

  $1\ g\ cm = 1\ g\ f \times 1\ cm = 980\ erg$
- Relationship between kg m and g cm

  $1\ kg\ m = 10^5\ g\ cm.$
- Work done by a variable force, $W = \int_{x_i}^{x_f} F(x)dx$



W = area under the force-displacement graph.

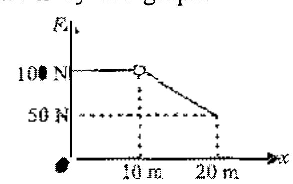
Illustration 1

A cyclist comes to a studding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposed to the motion. How much work does the road do on the cycle?

Soln. : $W = Fd\cos\theta = (200\ N)(10\ m)\cos 180° = -2000\ J.$

Illustration 2

A woman pushes a trunk on a railway platform that has a rough surface. She applies a force F over a distance of 20 m as shown by the graph.



Calculate the work done by the woman.

Soln. : $W = \int Fdx$ = Area under the graph

W = Area of rectangle + Area of trapezium

$= (100\ N)(10\ m) + \frac{1}{2}(100\ N + 50\ N)(10\ m)$

$= 1000\ J + 750\ J = 1750\ J.$

Illustration 3

Under the action of a force, a 3 kg body moves such that $x = \frac{t^2}{2}$, where position x is in metre and t is in second. What is the work done by the force in first 3 second?

Soln. : $x = \frac{t^2}{2}$

$$v = \frac{dx}{dt} = \frac{2t}{2} = t;\ \left(\frac{dx}{dt}\right)_{t=0} = 0 \text{ and } \left(\frac{dx}{dt}\right)_{t=3} = 3\ m\ s^{-1}$$

$$W = \Delta k = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2} \times 3 \times [3^2 - 0^2] = 13.5\ J.$$