ENERGY

It is the capacity of doing work.

- Energy is a scalar quantity. Its dimensional formula and S.I. unit is same as that of work.
- Some practical units of energy and their equivalence to joule are given in the table below.

S. No	Unit	Symbol	Equivalence in (J)
1.	1 erg	erg	10^{-7} J
2.	1 calorie	cal	4.2 J
3.	1 kilowatt hour	kWh	3.6×10^{6} J
4.	1 electron volt	eV	1.6×10^{-19} J

Kinetic Energy

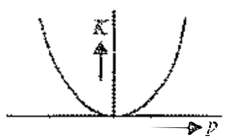
It is defined as the energy possessed by a body by virtue of its motion. It is generally represented by the letter K. Kinetic energy of a body of mass m moving with velocity v is given by

$$K = \frac{1}{2}mv^2.$$

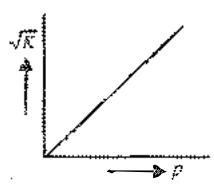
- Relation between kinetic energy (K) and linear momentum (p)

$$K = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2mK}$$

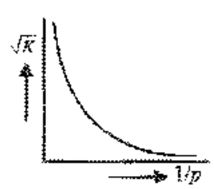
- The graph between K and p is a parabola as shown in the figure.



- The graph between $\sqrt{K}$ and p is a straight line as shown in figure.



- The graph between $\sqrt{K}$ and $1/p$ is a rectangular hyperbola as shown in figure.



- **Work energy theorem :** It states that work done by a force acting on a body is equal to change in kinetic energy of the body.

$$W = K_f - K_i$$

where K_i and K_f are initial and final kinetic energies of the body.

Illustration 4

A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is

(a) 0.5 J (b) –0.5 J
(c) –1.25 J (d) 1.25 J.

Soln. : (c) Kinetic energy at projection point is converted into potential energy of the particle during rise. Potential energy measures the work done against the force of gravity during rise.

$\therefore$ (– work done) = Kinetic energy = $\frac{1}{2}mv^2$

or (– work done) $= \frac{1}{2} \times \left(\frac{100}{1000}\right)(5)^2 = \frac{5 \times 5}{2 \times 10} = 1.25$ J

$\therefore$ Work done by force of gravity = – 1.25.

CONSERVATIVE AND NON CONSERVATIVE FORCES

- **Conservative force :** A force is said to be conservative if the work done by the force on a body is path independent and depends only on the initial and final positions. Equivalently a force is said to be conservative if the work done by it in moving a body around a closed path is zero.

  Gravitational force, electrostatic force and force in an elastic spring are conservative forces. All central forces are conservative forces.
- **Non-conservative forces :** A force is said to be non-conservative if the work done by the force on a body is path dependent. The work done by such a force in moving a body around a closed path is not zero. Frictional and viscous forces are non-conservative forces.

Potential Energy

- It is defined as the energy possessed by the body by virtue of its position or configuration. It is generally represented by the letter U.
- **Elastic potential energy :** It is energy associated with state of compression or stretching of an elastic spring and is given by $U = \frac{1}{2}kx^2$

  where k is the spring constant and x is the stretch or compression.
- **Gravitational potential energy :** It is the energy associated with two bodies of masses m_1 and m_2 separated by distance r and is given by $U = -\frac{Gm_1m_2}{r}$

  For a body of mass m at height h relative to surface of earth this potential energy reduces to

$$U = \frac{mgh}{\left(1 + \frac{h}{R}\right)}$$

  where R is the radius of earth.

  If $h << R$, $(h/R) << 1$. So $U = mgh$.
- Conservative force is the negative gradient of potential energy.

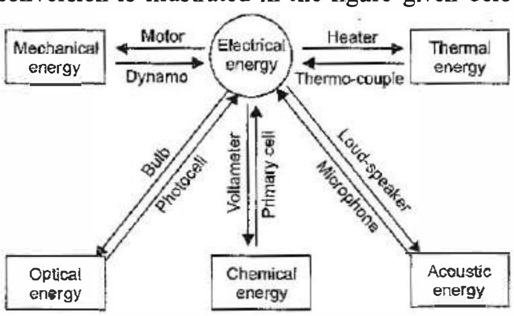
$$\vec{F} = -\left(\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right) = -\vec{\nabla} U$$

where $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

In one dimension, $\vec{F} = -\frac{dU}{dx}$ or $U = -\int F dx$

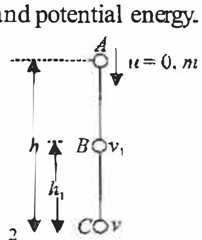
Mechanical Energy

- It is defined as the sum of kinetic energy K and potential energy U.
  i.e. $E = K + U$.
- **Principle of conservation of mechanical energy :** It states that for conservative forces the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depend on time.
  i.e. $K_1 + U_1 = K_2 + U_2$ or $K + U =$ constant
- **Law of conservation of energy :** It states that energy may be transformed from one form to another but it can neither be created nor be destroyed. The total energy of an isolated system remains constant.
- Conversion of electrical energy to various forms of energy or vice versa along with devices used for conversion is illustrated in the figure given below:



Examples of Conservation of Mechanical Energy

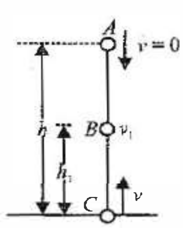
- **Freely falling body :** At the maximum height, total energy is in the form of potential energy. In the middle, total energy is in the form of both kinetic and potential energy. At the lowest point, total energy is in the form of kinetic energy



$\therefore$ $E = U_A = K_B + U_B = K_C$

or $E = mgh = \frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv^2$

- **Body projected vertically upwards :** At the lowest point energy is only kinetic, in the middle energy is both kinetic and potential and at the highest point, energy is only potential.



$\therefore$ $E = K_C = K_B + U_B = U_A$

or $E = \frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + mgh_1 = mgh$

Illustration 5

A uniform chain of length L and mass m is lying on a smooth table. One-third of its length is hanging vertically down over the edge of the table. How much work need to be done to pull the hanging part back to the table?

Soln. : Mass of hanging part of chain = $m/3$

Position of centre of gravity below table = $L/6$

$\therefore$ Work done = Potential energy change

or Work done = $\left(\frac{m}{3}\right)g\left(\frac{L}{6}\right) = \frac{mgL}{18}$

or Work required to be done = $\frac{mgL}{18}$.

Illustration 6

A particle is moving in a potential region defined by $U = K(x^2 + y^2 + z^2)$. Calculate the force acting on the particle.

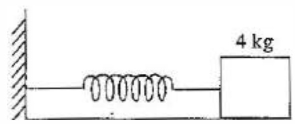
Soln. : $\vec{F} = -\hat{i}\frac{\partial U}{\partial x} - \hat{j}\frac{\partial U}{\partial y} - \hat{k}\frac{\partial U}{\partial z}$

or $\vec{F} = -K[\hat{i} \times 2x + \hat{j} \times 2y + \hat{k} \times 2z]$

or $\vec{F} = -2K[x\hat{i} + y\hat{j} + z\hat{k}]$

Illustration 7

A block of mass 4 kg while at rest is attached to an unstretched spring. The force constant of spring is 24 N m^{-1}. If a constant horizontal force of 10 N is applied on the block, the spring gets compressed by 0.5 m. What is the speed of the block at this point?



Soln. : Given: $k = 24$ N m^{-1}, $m = 4$ kg, $S = 0.5$ m work done by the applied force provides kinetic energy to mass and elastic potential energy to the spring.

$\therefore$ $10 \times 0.5 = \left(\frac{1}{2} \times 4 \times v^2\right) + \left[\frac{1}{2} \times 24 \times (0.5)^2\right]$

or $5 = 2v^2 + 3$ or $2v^2 = 2$

or $v = 1$ m s^{-1}

Speed of the block $= 1$ m s^{-1}.

- **Mass energy equivalence :** Albert Einstein showed that mass and energy are equivalent and are related by the relation $E = mc^2$
  where c is the speed of light in vacuum.

POWER

- It is defined as the rate of doing work.
- **Instantaneous power,** $P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$

  where $\vec{v}$ is the instantaneous velocity when the force is $\vec{F}$.

- **Average power :** It is defined as the ratio of the work ΔW, to the time Δt, *i.e.*, $P_{av} = \Delta W/\Delta t$.
- Power is a scalar quantity. Its dimensional formula is $[ML^2T^{-3}]$.
- **Units of power :** In SI system the absolute unit of power is watt. It is denoted by symbol W.
  $1 \text{ watt} = 1 \text{ J s}^{-1}$.
- In CGS system, the absolute unit of power is erg s^{-1}.
  $1 \text{ W} = 10^7 \text{ erg s}^{-1}$.
  Bigger units of power are
  - 1 kilowatt = 1 kW = 10^3 W
  - 1 megawatt = 1 MW = 10^6 W
- In engineering horse power is the practical unit of power.
  1 hp = 746 W.

Illustration 8

A body of mass m, accelerates uniformly from rest to v_1 in time t_1. Find the instantaneous power delivered to the body as a function of time t.

Soln. : Acceleration, $a = \dfrac{v_1}{t_1}$

$\therefore$ velocity, $(v) = 0 + at = \dfrac{v_1}{t_1}t$

$\therefore$ Power, P = Force × velocity = $m\,a\,v$

or $P = m\left(\dfrac{v_1}{t_1}\right) \times \left(\dfrac{v_1 t}{t_1}\right) = \dfrac{m v_1^2 t}{t_1^2}$.

Illustration 9

An elevator can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2 m s^{-1}. The frictional force opposing the motion is 4000 N. Determine the minimum horse power delivered by motor to the elevator.

Soln.: The downward force on the elevator is

$F = mg + F_f = (1800 \times 10) + 4000 = 22000$ N.

The motor must supply enough power to balance this force.

Hence, $P = \vec{F}.\vec{v} = 22000 \times 2 = 44000$ W.

COLLISION

- In physics a collision will take place if either of the two bodies come in physical contact with each other or even when path of one body is affected by the force exerted due to the other.
- Collisions are broadly classified into two types :

(i) **Elastic collision :** A collision in which both the momentum and kinetic energy of the body remains conserved. e.g. the collision between two glass balls.

The basic characteristics of an elastic collision are
- Momentum is conserved.
- Total energy is conserved.
- Kinetic energy is conserved.
- Forces involved in the interaction are conservative.
- Mechanical energy is not transformed into any other form of energy.

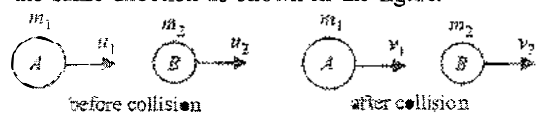
(ii) **Inelastic collision :** A collision in which only the momentum of the system is conserved but kinetic energy is not conserved. Most of the collisions in our day to day life are inelastic collisions.

e.g. mud thrown on the wall.

The basic characteristics of an inelastic collision are
- Momentum is conserved.
- Total energy is conserved.
- Kinetic energy is not conserved.
- Some or all of the forces involved are non-conservative.
- A part or whole of the mechanical energy may be transformed into other forms of energy.

Elastic Collision in one Dimension

- Consider two bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively. Assume that $u_1 > u_2$ so that two bodies collide. Let v_1 and v_2 be the final velocities of the bodies after collision. The two bodies suffer head on collision and continue moving along the straight line in the same direction as shown in the figure.



Then, $$v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)} + \frac{2m_2u_2}{m_1 + m_2} \quad ...(i)$$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)}{m_1 + m_2}u_2 \quad ...(ii)$$

- **Special cases**
  - When masses of two bodies are equal, *i.e.* $m_1 = m_2 = m$.
    From (i), we get $v_1 = u_2$.
    From (ii), we get $v_2 = \dfrac{2mu_1}{2m} = u_1$
    Thus, if two bodies of equal masses undergo elastic collision in one dimension, then after the collision, the bodies will exchange their velocities.
  - When the body B is initially at rest *i.e.*, $u_2 = 0$.
    From equation (i) and (ii), we get

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} \quad ...(iii)$$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2} \quad ...(iv)$$

Three cases arise further :

(a) When masses of two bodies are equal

i.e. $m_1 = m_2 = m$

From (iii), we get $v_1 = 0$

From (iv), $v_2 = u_1$

Therefore, when body A collides with body B of equal mass at rest, the body A comes to rest while the body B moves on with the velocity of the body A.

In this case, transfer of kinetic energy is hundred percent.