- **Average power :** It is defined as the ratio of the work ΔW, to the time Δt, *i.e.*, $P_{av} = \Delta W/\Delta t$.
- Power is a scalar quantity. Its dimensional formula is $[ML^2T^{-3}]$.
- **Units of power :** In SI system the absolute unit of power is watt. It is denoted by symbol W.

  1 watt = 1 J s^{-1}.
- In CGS system, the absolute unit of power is erg s^{-1}.

  1 W = 10^7 erg s^{-1}.

  Bigger units of power are
  - 1 kilowatt = 1 kW = 10^3 W
  - 1 megawatt = 1 MW = 10^6 W
- In engineering horse power is the practical unit of power.

  1 hp = 746 W.

Illustration 8

A body of mass m, accelerates uniformly from rest to v_1 in time t_1. Find the instantaneous power delivered to the body as a function of time t.

Soln. : Acceleration, $a = \frac{v_1}{t_1}$

$\therefore$ velocity, $(v) = 0 + at = \frac{v_1}{t_1}t$

$\therefore$ Power, P = Force × velocity = $m\,a\,v$

or $P = m\left(\frac{v_1}{t_1}\right) \times \left(\frac{v_1 t}{t_1}\right) = \frac{mv_1^2 t}{t_1^2}$.

Illustration 9

An elevator can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2 m s^{-1}. The frictional force opposing the motion is 4000 N. Determine the minimum horse power delivered by motor to the elevator.

Soln.: The downward force on the elevator is

$F = mg + F_f = (1800 \times 10) + 4000 = 22000$ N.

The motor must supply enough power to balance this force. Hence, $P = \vec{F}.\vec{v} = 22000 \times 2 = 44000$ W.

COLLISION

- In physics a collision will take place if either of the two bodies come in physical contact with each other or even when path of one body is affected by the force exerted due to the other.
- Collisions are broadly classified into two types :

(i) **Elastic collision :** A collision in which both the momentum and kinetic energy of the body remains conserved. e.g. the collision between two glass balls.

The basic characteristics of an elastic collision are
- Momentum is conserved.
- Total energy is conserved.
- Kinetic energy is conserved.
- Forces involved in the interaction are conservative.
- Mechanical energy is not transformed into any other form of energy.

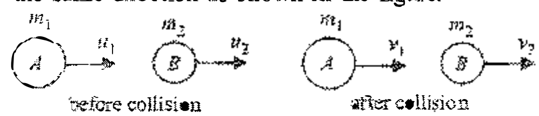
(ii) **Inelastic collision :** A collision in which only the momentum of the system is conserved but kinetic energy is not conserved. Most of the collisions in our day to day life are inelastic collisions.

e.g. mud thrown on the wall.

The basic characteristics of an inelastic collision are
- Momentum is conserved.
- Total energy is conserved.
- Kinetic energy is not conserved.
- Some or all of the forces involved are non-conservative.
- A part or whole of the mechanical energy may be transformed into other forms of energy.

Elastic Collision in one Dimension

- Consider two bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively. Assume that $u_1 > u_2$ so that two bodies collide. Let v_1 and v_2 be the final velocities of the bodies after collision. The two bodies suffer head on collision and continue moving along the straight line in the same direction as shown in the figure.



Then, $$v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)} + \frac{2m_2u_2}{m_1 + m_2} \quad ...(i)$$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)}{m_1 + m_2}u_2 \quad ...(ii)$$

- **Special cases**
  - When masses of two bodies are equal, *i.e.* $m_1 = m_2 = m$.

    From (i), we get $v_1 = u_2$.

    From (ii), we get $v_2 = \frac{2mu_1}{2m} = u_1$

    Thus, if two bodies of equal masses undergo elastic collision in one dimension, then after the collision, the bodies will exchange their velocities.
  - When the body B is initially at rest *i.e.*, $u_2 = 0$. From equation (i) and (ii), we get

    $$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} \quad ...(iii)$$

    $$v_2 = \frac{2m_1u_1}{m_1 + m_2} \quad ...(iv)$$

    Three cases arise further :

    (a) When masses of two bodies are equal *i.e.* $m_1 = m_2 = m$

    From (iii), we get $v_1 = 0$

    From (iv), $v_2 = u_1$

    Therefore, when body A collides with body B of equal mass at rest, the body A comes to rest while the body B moves on with the velocity of the body A.

    In this case, transfer of kinetic energy is hundred percent.

(b) When the body B has negligible mass as compared to that of body A *i.e.*, $m_2 << m_1$ then in equations (iii) and (iv), m_2 can be neglected as compared to m_1.

$$\therefore \quad v_1 = \frac{m_1 u_1}{m_1} = u_1,\ v_2 = \frac{2m_1 u_1}{m_1} = 2u_1$$

Therefore, when a heavy body A collides with a light body B at rest, the body A should keep on moving with same velocity and the body B starts moving with velocity double that of A.

(c) When a mass of body B is very large as compared to that of body A *i.e.*, $m_2 >> m_1$ then in equations (iii) and (iv), m_1 can be neglected as compared to m_2.

$$v_1 = -\frac{m_2 u_1}{m_2} = -u_1,\ v_2 = 0.$$

Therefore, when a light body A collides with a heavy body B at rest, the body A should start moving with same velocity just in opposite direction while the body B should practically remain at rest.

Illustration 10

A body of mass m collides elastically with a stationary body of mass M. After collision, m has a speed equal to one-third of its initial speed. Calculate the ratio (m/M).

Soln. : $v_1 = \dfrac{(m_1 - m_2)u_1}{(m_1 + m_2)}$ when $u_2 = 0$

$$\therefore \quad \frac{u_1}{3} = \frac{(m - M)u_1}{(m + M)}$$

$\therefore \quad 3m - 3M = m + M \qquad$ or $\quad 2m = 4M$

or $\quad \dfrac{m}{M} = \dfrac{4}{2} = \dfrac{2}{1}$.

Illustration 11

Two identical bodies A and B moving with velocities 13 m/s and – 15 m/s respectively collide head-on elastically. What will be their velocities after collision?

Soln. : When masses are equal, velocities are just interchanged after an elastic head-on collision.

$\therefore$ After collision, velocity of A = – 15 m/s

velocity of B = 13 m/s

Illustration 12

A massive ball moving with speed v collides with a tiny ball by negligible mass. The collision is elastic. Find the speed with which the second tiny ball will move.

Soln. : $v_2 = \dfrac{2m_1 u_1}{(m_1 + m_2)}$

Since $m_1 >> m_2$, $(m_1 + m_2) = m_1$.

or $\quad v_2 = \dfrac{2m_1 v}{m_1} = 2v$

$\therefore$ Speed of second tiny ball = $2v$.

Illustration 13

A neutron travelling with a velocity v and kinetic energy E collides head-on elastically with a nucleus of mass number A at rest. Calculate the fraction of total energy retained by the neutron.

Soln. : $v_1 = \dfrac{(m_1 - m_2)u_1}{(m_1 + m_2)}$ while $u_2 = 0$.

$$\therefore \quad v_1 = \frac{(1 - A)u_1}{(1 + A)}.$$

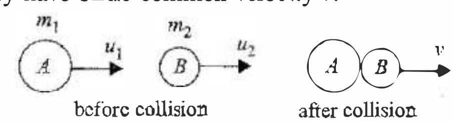
Neutron is ${}_0n^1$. Its mass number = 1.

Fraction of energy retained by neutron

$$= \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mu_1^2} = \left(\frac{v_1}{u_1}\right)^2 = \left(\frac{1-A}{1+A}\right)^2 = \left(\frac{A-1}{A+1}\right)^2$$

Perfectly Inelastic Collision in One Dimension

- Consider two bodies A and B of masses m_1 and m_2 moving with velocities u_1 and u_2 $(u_2 < u_1)$ respectively along the same line collide head on and after collision they have same common velocity v.



According to conservation of linear momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v + m_2 v$$

or $\quad v = \dfrac{m_1 u_1 + m_2 u_2}{(m_1 + m_2)} \qquad$ ...(i)

Kinetic energy of the system before collision is

$$K_I = \frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2$$

Kinetic energy of the system after collision is

$$K_F = \frac{1}{2}(m_1 + m_2)v^2$$

Loss in kinetic energy during collision,

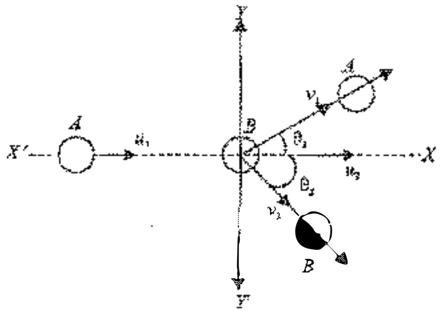
$$\Delta K = K_I - K_F = \left[\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2\right] - \frac{1}{2}(m_1 + m_2)v^2 \quad \text{...(ii)}$$

Substituting the value of v from eqn. (i), we get

$$\Delta K = \frac{1}{2}\frac{m_1 m_2}{(m_1 + m_2)}(u_1 - u_2)^2$$

Elastic Collision in Two Dimensions or Oblique Collision

- Let us consider two bodies A and B of masses m_1 and m_2 moving along X-axis with velocities u_1 and u_2 respectively. When $u_1 > u_2$, the two bodies collide. After collision, body A moves with velocity v_1 at an angle θ_1 with X-axis and body B moves with a velocity v_2 at an angle θ_2 with X-axis as shown in the figure.



- Since the collision is elastic, kinetic energy is conserved.

  $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad \text{...(i)}$$

  Also momentum along X-axis before collision = momentum after collision along X-axis

  $$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$$

  Similarly along Y-axis

  $$0 = m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2 \qquad \text{...(ii)}$$

  Thus from these three equation (i), (ii) and (iii) we can find the required quantities.

Illustration 14

A block of mass 0.50 kg is moving with a speed of 2.00 m s^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. What is the energy loss during the collision?

Soln. : By the law of conservation of momentum $mu = (M + m)v$

$$0.50 \times 2.00 = (1 + 0.50)v, \quad \frac{1.00}{1.50} = v$$

Initial K.E. = $(1/2) \times 0.50 \times (2.00)^2 = 1.00$ J.

$$\text{Final K.E.} = \frac{1}{2} \times 1.50 \times \frac{1.00^2}{(1.50)^2} = \frac{1.00}{3.00} = 0.33$$

$\therefore$ Loss of energy = 1.00 – 0.33 = 0.67 J.

Coefficient of Restitution

- It is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is represented by e.

  $$e = \frac{\text{relative velocity of separation (after collision)}}{\text{relative velocity of approach (before collision)}}$$

  $$e = \frac{v_2 - v_1}{u_1 - u_2}$$

  where u_1, u_2 are velocities of two bodies before collision, and v_1, v_2 are their respective velocities after collision.
  - For perfectly elastic collision, $e = 1$.
  - For perfectly inelastic collision, $e = 0$.
- A ball falls from a height h, it strikes the ground with a velocity $u = \sqrt{2gh}$. Let it rebound with a velocity v and rise to a height h_1. Then,

  $$e = \frac{v}{u} = \frac{\sqrt{2gh_1}}{\sqrt{2gh}} = \sqrt{\frac{h_1}{h}} \quad \text{or} \quad \sqrt{h_1} = e\sqrt{h} \quad \text{or} \quad h_1 = e^2h.$$

- A ball dropped from a height h and attaining height h_n after n rebounds. Then $h_n = e^{2n}h$.
- A ball dropped from height h and travelling a total distance S before coming to rest.

  Then $S = h\left[\frac{1+e^2}{1-e^2}\right]$

- A ball dropped from a height h and rebounding. The time taken by the ball in rising to height h_1 and coming back is

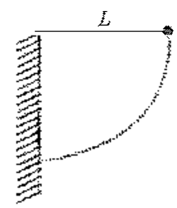
  $$2\sqrt{\frac{2h_1}{g}} = 2e\sqrt{\frac{2h}{g}}.$$

  Note that $h_1 = e^2h$.

  Total time taken by the ball in coming to rest is

  $$t = \frac{1+e}{1-e}\sqrt{\frac{2h}{g}}.$$

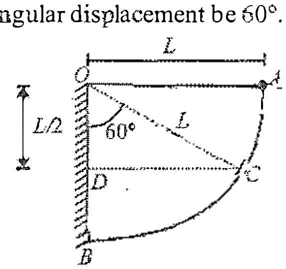
Illustration 15



A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position (see fig.) and released. The ball hits the wall, the co-efficient of restitution being $\frac{2}{\sqrt{5}}$.

What is the minimum number of collisions after which the amplitude of oscillations becomes less than 60 degree?

Soln : The bob is released from horizontal position OA. It travels along ACB. At C, let the angular displacement be 60°.



$$OD = L\cos 60° = \frac{L}{2}.$$

Velocity at B when the ball starts from $A = v_B$.

$$\therefore mgL = \frac{1}{2}mv_B^2$$

$$\text{or} \quad v_B = \sqrt{2gL}$$

The velocity at B when the ball starts from $C = v'_B$

$$\therefore mg\frac{L}{2} = \frac{1}{2}m(v'_B)^2 \Rightarrow v'_B = \sqrt{gL}$$

It means that the velocity of bob is reduced from $\sqrt{2gL}$ to $\sqrt{gL}$ due to collisions with wall. Let e denote co-efficient of restitution.

Let n = number of collisions.

$$\therefore e^n\left(\sqrt{2gL}\right) = \sqrt{gL} \quad \text{or} \quad \left(\frac{2}{\sqrt{5}}\right)^n \times \sqrt{2} = 1$$

$$\text{or} \quad n\log 2 - \frac{n}{2}\log 5 + \frac{1}{2}\log 2 = 0$$

By solving, we get $n = 3.1$

$\therefore$ Number of collision = 4.