CONCEPT MAP

Motion of System of Particle and Rigid Body

entre of mass : Centre of ass of a system is defined as at point where whole mass the system is supposed to concentrated for the anslational motion of the stem.

Two particle system : The position vector of CM of two particle

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

***n* particle system :** In case of n particle system, the position vector of CM is

$$\vec{R} = \frac{\sum_{i=1}^{i=n} m_i\vec{r}_i}{\sum_{i=1}^{i=n} m_i}$$

Velocity of centre of mass of a system of particles, $\vec{v}_{cm} = \frac{d\vec{R}}{dt}$

Acceleration of centre of mass of a system of particles, $\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt}$

Sum of all external forces acting on the system of particles, $\vec{F}_{ext} = M\,\vec{a}_{cm}$

Equations of rotational motion :

- $\omega = \omega_0 + \alpha t$
- $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
- $\omega^2 - \omega_0^2 = 2\alpha\theta$

Law of conservation of angular momentum: If no external torque acts on a system, the total angular momentum of the system remains conserved.

If $\vec{\tau}_{ext} = 0;\ \frac{d\vec{L}}{dt} = 0$

$\therefore\ \vec{L} =$ constant

Also, $I_1\omega_1 = I_2\omega_2$

ngular velocity : It is the time ate of change of angular isplacement, $\vec{\omega} = \frac{d\theta}{dt}$

$\vec{v} = \vec{\omega} \times \vec{r}$

Angular acceleration : It is the time rate of change of angular

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt},\quad \vec{a} = \vec{\alpha} \times \vec{r}$$

Angular momentum : It is the product of linear momentum and perpendicular distance of line of action of linear momentum vector from the axis of rotation, in vector form

$$\vec{L} = \vec{r} \times \vec{p}$$

Moment of force or Torque : It is equal to the product of force and perpendicular distance

$\vec{\tau} = \vec{r} \times \vec{F};\ |\vec{\tau}| = rF\sin\theta = rF_\perp = r_\perp F$

Relation between torque and angular acceleration

$$\vec{\tau} = I\vec{\alpha}$$

Moment of inertia : It is the uantity that measures the inertia of rotational motion of the body. For a particle, $I = mr^2$

$I = \int r^2 dm$

Unit : kg m^2

Radius of gyration : It is equal to the root mean square distance of the constituent particles of the body from the given axis,

$$K = \sqrt{\frac{r_1^2 + r_2^2 + \ldots + r_n^2}{n}},\quad I = MK^2$$

Theorem of parallel axes

$I = I_{cm} + mh^2$

Theorem of perpendicular axes

$I_z = I_x + I_y$

Kinetic energy and M.I. : The energy possessed by the body on account of its rotation about a given axis is the kinetic energy of rotation

i.e., K.E. $= \frac{1}{2}I\omega^2$

Work done by torque τ in rotating a body by an angle θ

$$= \int_0^\theta \tau\, d\theta$$

Instantaneous power, $P = \tau\omega$

Equilibrium of rigid body :

- Net external force acting on the body must be zero

  i.e., $\sum \vec{F}_{ext} = 0$

- Net external torque on the

  i.e., $\vec{\tau}_{ext} = 0$

Symbols Used

m_1, m_2 = mass of particles
$\vec{R}$ = position vector of centre of mass
$\vec{r}_1, \vec{r}_2$ = position vector of particles
M = total mass
$\vec{\omega}$ = angular velocity
$\vec{\theta}$ = angular displacement
$\vec{\tau}$ = moment of force or torque
$\vec{L}$ = angular momentum
$\vec{p}$ = linear momentum
$\vec{\omega}_0$ = initial angular velocity
$\vec{\alpha}$ = angular acceleration
K = radius of gyration
n = number of particles
I = moment of inertia
I_{cm} = moment of inertia about centre of mass
I_x, I_y, I_z = moment of inertia about x, y and z axes respectively
M.I. = moment of inertia