UNIT 6

Gravitation

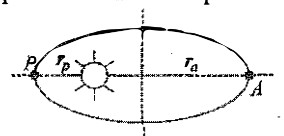
KEPLER'S LAWS OF PLANETARY MOTION

Kepler's three laws of planetary motion are as follows :

- **Kepler's first law (law of orbits) :** All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.
- **Kepler's second law (law of areas) :** The radius vector drawn from the sun to the planet sweeps out equal areas in equal intervals of time *i.e.* the areal velocity of the planet (or the area swept out by the planet per unit time) around the sun is constant *i.e.* areal velocity $=\frac{d\vec{A}}{dt}=$ a constant, for a planet.
  - Angular momentum $(\vec{L})$ of a planet is related with areal velocity $\left(\frac{d\vec{A}}{dt}\right)$ by the relation
  $$\vec{L}=2m\left(\frac{d\vec{A}}{dt}\right)$$
  - Kepler's second law follows from the law of conservation of angular momentum.
  - The area covered by the radius vector in dt seconds $=\frac{1}{2}r^2\,d\theta$.
  - The areal velocity $=\frac{1}{2}r^2\frac{d\theta}{dt}=\frac{1}{2}r^2\omega=\frac{1}{2}rv$.
  - According to Kepler's second law, the speed of the planet is maximum, when it is closest to the sun and is minimum when the planet is farthest from the sun.
- **Kepler's third law (Law of periods) :** The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semi major axis of the elliptical orbit *i.e.* $T^2 \propto a^3$ where a is the semi major axis of the elliptical orbit of the planet around the sun.

Illustration 1

Let the speed of the planet at the perihelion P be v_p and the sun-planet distance be r_p. Relate (r_p, v_p) to the corresponding quantities at the aphelion (r_a, v_a)



(a) $\frac{r_p}{r_a}=\frac{v_p}{v_a}$ (b) $\frac{r_p^2}{r_a^2}=\frac{v_a}{v_p}$

(c) $\frac{v_p}{v_a}=\frac{r_a}{r_p}$ (d) $\frac{v_p^2}{v_a^2}=\frac{r_a}{r_p}$

Soln. (a) : At the perihelion r_p and v_p are mutually perpendicular so r_a is normal to v_a. The angular momentum is conserved,

$$\therefore\ m\,v_a r_a = m v_p r_p \Rightarrow \frac{v_p}{v_a}=\frac{r_a}{r_p}$$

Illustration 2

The rotation period of an Earth satellite close to the surface of the Earth is 83 minutes. Find the time period of another earth satellite in an orbit at a distance of three earth radii from its surface

Soln. : $T^2 \propto R^3$

$$\therefore\ \left(\frac{T_2}{T_1}\right)^2=\left(\frac{R_2}{R_1}\right)^3 \Rightarrow \left(\frac{T_2}{83\text{ min}}\right)^2=\left(\frac{R+3R}{R}\right)^3$$

$\Rightarrow$ $T_2 = 664$ min.

Illustration 3

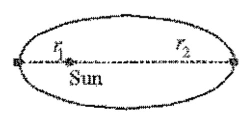
A planet of mass m moves around the sun of mass M in an elliptical orbit. The maximum and minimum distances of the planet from the Sun are r_1 and r_2 respectively. Find the time period of the planet.

Soln. : According to Kepler's 3rd law, $T^2 \propto a^3$

where a is the semi-major axis, Here,

$$a=\left(\frac{r_1+r_2}{2}\right) \Rightarrow T^2 \propto \left(\frac{r_1+r_2}{2}\right)^3$$

or $T \propto (r_1+r_2)^{\frac{3}{2}}$



NEWTON'S LAW OF GRAVITATION

- According to Newton's law of gravitation every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The direction of this force is along the line joining the two bodies.
- The magnitude of gravitational force acting between two bodies of masses m_1 and m_2 placed distance r apart is