In case of circular orbit, radius r is constant.

$\therefore$ mv = constant

$\therefore$ Linear momentum (p) is also conserved.

ESCAPE SPEED

- The escape speed on earth (or any planet) is defined as the minimum speed with which a body has to be projected vertically upwards from the surface of earth (or any other planet) so that it just crosses the gravitational field of earth (or of that planet) and never returns on its own. Escape speed v_e is given by

$$v_e = \sqrt{\frac{2GM}{R}}$$

where M = Mass of the earth/planet

R = Radius of the earth/planet

$$v_e = \sqrt{\frac{2G \times \text{volume} \times \text{density}}{R}}$$

or $v_e = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho} = \sqrt{\frac{8\pi\rho G R^2}{3}}$

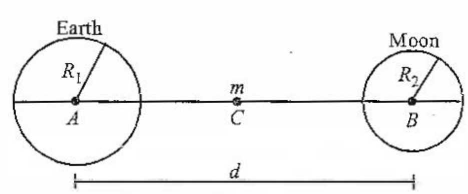
- The escape speed depends upon the mass and radius of the earth/planet from the surface of which the body is to be projected.
- The escape speed is independent of the mass and direction of projection of the body from the surface of earth/planet.
- For earth, $v_e = 11.2 \text{ km s}^{-1}$
- For a point close to earth's surface the escape speed and orbital speed are related as

$$v_e = \sqrt{2}\, v_o$$

- A given planet will have atmosphere if the root mean square speed of molecules in its atmosphere (*i.e.*, $v_{rms} = \sqrt{3RT/M}$) is smaller than the escape speed for that planet.
- Moon has no atmosphere because the r.m.s. speed of gas molecules there, are greater than the escape speed of moon.

Illustration 15

The masses and the radii of earth and moon are M_1, R_1 and M_2, R_2 respectively. Their centres are at distance d apart as shown in figure. Find the minimum speed with which a particle of mass m should be projected from a point midway between the two centres so as to escape to infinity.



Soln. : Let the potential energy of particle = E_p

$$\therefore \quad E_p = \frac{-GM_1m}{(d/2)} + \frac{-GM_2m}{(d/2)} = -\frac{2Gm(M_1+M_2)}{d}$$

Let the escape velocity of the particle = v_e

$\therefore$ Kinetic energy of particle $E_K = \frac{1}{2}mv_e^2$

Total energy = $E_K + E_p$

or $E = \frac{1}{2}mv_e^2 - \frac{2Gm(M_1+M_2)}{d}$

When the particle escapes to infinity, its total energy E becomes zero.

$$\therefore \quad \frac{1}{2}mv_e^2 - \frac{2Gm(M_1+M_2)}{d} = 0$$

or $v_e^2 = \frac{4G(M_1+M_2)}{d}$ or $v_e = 2\sqrt{\frac{G(M_1+M_2)}{d}}$.

Illustration 16

The escape velocity of a body from surface of earth is 11.2 km/s. A body is projected with a velocity of 22.4 km/s. Calculate the velocity of body at infinite distance from centre of earth.

Soln.: Let u = velocity of projection of the body

v = velocity of the body at infinite distance

$$\therefore \quad \frac{1}{2}mu^2 = \frac{1}{2}mv_e^2 + \frac{1}{2}mv^2$$

where v_e = velocity of escape.

or $v^2 = u^2 - v_e^2$

or $v = \sqrt{u^2 - v_e^2} = \sqrt{(22.4)^2 - (11.2)^2}$

$= (11.2)\sqrt{4-1} = 11.2 \times \sqrt{3}$

$\therefore$ Velocity of body at infinity $= 11.2\sqrt{3}$ km s^{-1}.

Illustration 17

A spaceship is launched into a circular orbit close to earth's surface. What additional velocity has now to be imparted to the spaceship in the orbit to overcome the gravitational pull? Take radius of earth = 6400 km, $g = 9.8 \text{ m s}^{-2}$

Soln.: Let v_o and v_e denote the orbital velocity and escape velocity of the spaceship.

$\therefore$ $v_o = \sqrt{gR} = \sqrt{9.8 \times 6400 \times 1000} = 8$ km/s

and $v_e = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6400 \times 1000} = 11.2$ km/s

$\therefore$ Additional velocity required = $v_e - v_o$

$\therefore$ $\Delta v = \sqrt{2gR} - \sqrt{gR} = (11.2 - 8.0)$ km/s = 3.2 km/s.