- The latent heat of a sustance is given by
$$L = \frac{Q}{m}$$
where m is the mass of a substance.
- The SI unit of latent heat is J kg^{-1} while practical unit is cal g^{-1}.
- **Latent heat of fusion :** It is the amount of heat required to change unit mass of the solid into liquid at constant temperature e.g. latent heat of fusion of ice = 80 cal g^{-1}.
- **Latent heat of vaporisation :** It is the amount of heat required to change unit mass of the liquid into vapour at constant temperature. e.g. latent heat of vaporisation of water = 540 cal g^{-1}.

PRINCIPLE OF CALORIMETRY

- When two bodies at different temperatures are placed in contact with each other then heat will pass from the body at higher temperature to the body at lower temperature until both reach a common temperature.
  i.e. heat lost by one body = heat gained by the other.
- Principle of calorimetry obeys law of conservation of energy.

Illustration 19

A Centigrade and a Fahrenheit thermometer are dipped in boiling water. The water temperature is lowered until the Fahrenheit thermometer registers 140°. What is the fall in temperature as registered by the Centigrade thermometer?

(a) 30° (b) 40°
(c) 60° (d) 80°

Soln. (b) : The relation between Centigrade and Fahrenheit scale is as follows:

$$C = \frac{5}{9}(F - 32)$$

Here the Fahrenheit temperature is 140°F, so

$$T_{final} = \frac{5}{9}(140 - 32) \Rightarrow T_{final} = 60°C$$

The fall in temperature from the boiling point
= 100°C − 60°C = 40°C.

⇒ (b) is correct.

Illustration 20

Two liquids A and B are at 32°C and 24°C. When mixed in equal masses, the temperature of mixture is found to be 28°C. Their specific heats are in the ratio of

(a) 3 : 2 (b) 2 : 3
(c) 1 : 1 (d) 4 : 3

Soln. (c) : When mixed, the heat gained by liquid B is equal to heat lost by liquid A.

The final temperature of the mixture is 28°C. Heat lost by
$A = m_A s_A (32 - 28)$

Heat gained by $B = m_B s_B (28 - 24)$

$m_A s_A(32 - 28) = m_B s_B(28 - 24)$

As $m_A = m_B$, $\Rightarrow s_A = s_B$ or $s_A : s_B = 1 : 1$

⇒ (c) is correct.

Illustration 21

A metallic rod 1 cm long and 1 sq. cm in cross-section is heated to t°C. If the linear expansion is α per degree centigrade and Y is the Young's modulus, the force required to prevent the rod from expanding is

(a) $Y\alpha t$ (b) $\frac{Y\alpha t}{1+\alpha t}$

(c) $\frac{Y\alpha t}{1-\alpha t}$ (d) None of these

Soln. (a) : If the rod is heated, thermal expansion
$\Delta l = \alpha \cdot l \cdot \Delta T = \alpha \cdot (1 \text{ cm}) (t°C)$

If the rod is not allowed to expand by applying a force, it means a mechanical compression equal to the thermal expansion has been produced by applying the force.

Now, $Y = \left(\frac{F}{A}\right) \div \left(\frac{\Delta l}{l}\right)$, $Y = \frac{F \cdot l}{A \Delta l}$ or $F = YA\left(\frac{\Delta l}{l}\right)$

$F = YA(\alpha t)$ As $A = 1$ cm²

$F = Y\alpha t \Rightarrow$ (a) is correct.

Illustration 22

The absolute coefficient of expansion of a liquid is 7 times that of the volume coefficient of expansion of the vessel. Then the ratio of absolute and apparent expansion of the liquid is

(a) $\frac{1}{7}$ (b) $\frac{7}{6}$ (c) $\frac{6}{7}$ (d) none of these

Soln. (b) : Given: $\gamma_{liquid} = 7\gamma_{vessel}$

$\Delta V_{Absolute} = \gamma_{liquid} \cdot V\, \Delta T = 7\gamma_{vessel} \cdot V \Delta T$ ...(i)

$\Delta V_{vessel} = \gamma_{vessel} \cdot V \cdot \Delta T$

$\Delta V_{Apparent} = \Delta V_{liquid} - \Delta V_{vessel}$

$= V \cdot \Delta T(\gamma_{liquid} - \gamma_{vessel}) = V \cdot \Delta T(7\gamma_{vessel} - \gamma_{vessel})$

$\Rightarrow \Delta V_{Apparent} = V \cdot \Delta T(6\gamma_{vessel})$...(ii)

Equation (1) divided by equation (2), we get,

$$\frac{\Delta V_{Absolute}}{\Delta V_{Apparent}} \quad \frac{\gamma_{Absolute}}{\gamma_{Apparent}} = \frac{7\gamma_{vessel} \cdot V \Delta T}{6\gamma_{vessel} \cdot V \Delta T} = \frac{7}{6}$$

⇒ (b) is correct.

HEAT TRANSFER

- The three modes of heat transfer are conduction, convection and radiation.
- **Conduction :** It is the process in which heat is transmitted from one point to the other through the substance without the actual motion of the particles.
- **Convection :** It is the process in which heat is transmitted from one place to the other by the actual movement of the heated particles. It is prominent in the case of liquids and gases. Land and sea breezes and trade winds are formed due to convection. Convection plays an important role in ventilation, gas filled electric lamps and heating of buildings by hot water circulation.
- **Radiation :** It is the process in which heat is transmitted from one place to the other directly without the necessity of the intervening medium.
- Heat from the sun reaches the earth by radiation.

- Radiation is the fastest mode of heat transfer.
- When a bar of length L and uniform area of cross-section A with its ends maintained at temperatures T_1 and T_2, the rate of flow of heat (or heat current) H is given by

$$H = \frac{KA(T_1 - T_2)}{L}$$

  where K is the thermal conductivity of the material of the bar.
- Thermal conductivity depends upon the nature of material.
- The SI unit of thermal conductivity K is J $s^{-1}m^{-1}K^{-1}$ or W $m^{-1}K^{-1}$ while practical unit is cal/cm s °C. The dimensional formula of K is $[MLT^{-3}K^{-1}]$.

Wiedemann-Franz Law

- According to this law the ratio of the thermal and electrical conductivities is the same for all metals at the same temperature. Moreover, the ratio is directly proportional to the absolute temperature of the metal.

$$\frac{K}{\sigma} \propto T$$

  i.e., $\frac{K}{\sigma T}$ = a constant,

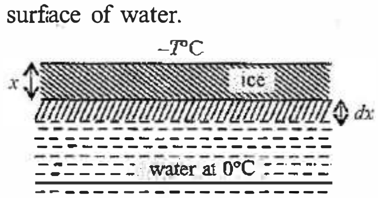
  where K = thermal conductivity, σ = electrical conductivity, T = the temperature in kelvin.
- Thermal resistance of the bar, $R_H = \frac{L}{KA}$.
- Comparison of heat conduction with electrical conduction

Electrical conduction	Thermal conduction
Electrical charge flows from higher potential to lower potential	Heat flows from higher temperature to lower temperature
The rate of flow of charge heat is called electric current	The rate of flow of is called as heat current
By Ohm's law, $I = \frac{V_1 - V_2}{R}$	Heat current $H = \frac{T_1 - T_2}{R_H}$
Electrical resistance, $R = \rho\frac{L}{A} = \frac{L}{\sigma A}$ where ρ is resistivity and σ is conductivity	Thermal resistance, $R_H = \frac{L}{KA}$ where K is thermal conductivity

- The flow of heat through rods in series and parallel is analogous to the flow of current through resistances in series and parallel.

Growth of Ice on Lakes

- The water in a lake freezes when the temperature of the air in contact with it falls below 0°C. The density of ice is less than that of water and so the ice formed floats on the surface of water.



Let x be the thickness of layer so formed. Let the temperature of air just above ice be $-T$°C and the temperature of water below ice in the lake be 0°C. The thickness of layer of ice will increase when heat from water below ice conducts through the ice layer already formed.

Let the cross-sectional area of the ice be A. Suppose the thickness of ice increases by dx in a time dt.

- Time taken by ice to grow a thickness x is given by

$$t = \frac{1}{2}\frac{\rho L x^2}{KT}$$

  where K is the thermal conductivity of ice, ρ is the density of ice, L is the latent heat of fusion of ice.
- Time taken to double and triple the thickness will be in the ratio

  $t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2$

  $t_1 : t_2 : t_3 :: 1 : 4 : 9$

STEFAN-BOLTZMANN LAW

- It states that the radiant energy emitted by a perfectly black body per unit area per sec (*i.e.*, radiancy or intensity of black body radiation) is directly proportional to the fourth power of its absolute temperature

  i.e. $E \propto T^4$ or $E = \sigma T^4$

  where σ is a constant called Stefan's constant.
- The value of σ in S.I. unit is 5.67×10^{-8} W m^{-2} K^{-4}.
- If the body is not a perfectly black body, then

$$E = e\sigma T^4$$

  where e is the emissivity of the body and has value $0 < e < 1$ depending upon the nature of surface. It has no units and dimensions.
- The energy radiated per second by a body of area $A = eA\sigma T^4$
- A body at temperature T with surroundings temperature T_S ($T > T_S$), the net rate of loss of radiant energy is

$$= eA\sigma(T^4 - T_S^4)$$

NEWTON'S LAW OF COOLING

- It states that the rate of cooling of a body is proportional to the excess temperature of the body over the surroundings.

$$\frac{dQ}{dt} = -k(T - T_S)$$

  where T_S is the temperature of the surrounding medium and T is the temperature of the body.

WIEN'S DISPLACEMENT LAW

- According to this law, the product of wavelength corresponding to maximum energy and the absolute temperature is constant.

  i.e., $\lambda_m T$ = constant
- The value of Wien's constant is 2.9×10^{-3} mK.
- The value of solar constant, S = 1340 Wm^{-2}.
- Temperature of sun is given by $T = \left(\frac{R^2 S}{R_S^2 \sigma}\right)^{1/4}$

  where R is the mean distance of the earth from the sun, R_s is the radius of the sun, S is solar constant and σ is Stefan's constant.

Illustration 23

Two slabs of thickness d_1 and d_2 and of conductivities K_1 and K_2 are placed in contact and combination behaves as a single slab of conductivity K given by

(a) $\frac{d_1}{K_1}$ (b) $d_1K_1 + d_2K_2 = K$

(c) $\frac{d_1}{K_1}+\frac{d_2}{K_2} = \frac{d_1+d_2}{K}$ (d) $K(d_1 + d_2) = K_1K_2d_1d_2$

Soln. (c) : Thermal resistances behave analogous to electrical resistances.

In series,

$$R_{eq} = R_1 + R_2 \quad \text{Now, } R_{thermal} = \frac{\Delta x}{KA}$$

Here, $\frac{(d_1+d_2)}{KA} = \frac{d_1}{K_1A} + \frac{d_2}{K_2A} \Rightarrow \frac{d_1}{K_1}+\frac{d_2}{K_2} = \frac{d_1+d_2}{K}$.

$\Rightarrow$ (c) is correct.

Illustration 24

A sphere is at a temperature 600 K. Its cooling rate is R in an external environment of 200 K. Its temperature falls to 400 K, then cooling rate R' will be

(a) $\frac{3}{16}R$ (b) $\frac{16}{3}R$

(c) $\frac{9}{27}R$ (d) None of these

Soln. (a) : From Stefan's law, net rate of heat energy lost per second

$R = \varepsilon\sigma A(T^4 - T_0^4)$

where T is the temperature of the body and T_0 is the temperature of the surroundings.

Here $R = \varepsilon\sigma A(600^4 - 200^4)$

$$R' = \varepsilon\sigma A(400^4 - 200^4) \Rightarrow \frac{R'}{R} = \left[\frac{400^4 - 200^4}{600^4 - 200^4}\right]$$

$$R' = \left(\frac{4^4-2^4}{6^4-2^4}\right)R$$

Hence, $R' = \frac{3}{16}R$. $\Rightarrow$ (a) is correct.

Illustration 25

A black body has maximum wavelength λ_m at 2000 K. Its corresponding wavelength at 3000 K will be

(a) $\frac{3}{2}\lambda_m$ (b) $\frac{2}{3}\lambda_m$ (c) $\frac{16}{81}\lambda_m$ (d) $\frac{81}{16}\lambda_m$

Soln. (b) : Using Wien's displacement law,

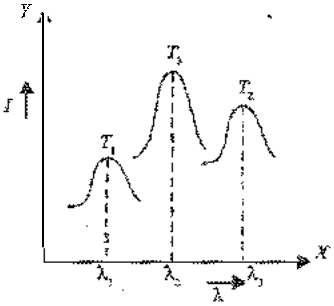
$\lambda_{max} \cdot T = \text{constant}$

Here $\lambda'_m \cdot (3000\text{ K}) = \lambda_m (2000\text{ K}) \Rightarrow \lambda'_m = \frac{2}{3}\lambda_m$.

$\Rightarrow$ (b) is correct.

Illustration 26

The plots of intensity versus wavelength for three black bodies at temperatures T_1, T_2 and T_3 respectively are shown. Which, of these temperatures are the lowest and the highest? Grade T_1, T_2 and T_3.



Soln.: According to Wien's law,

$$\lambda_m T = \text{constant} \Rightarrow T = \frac{\text{Constant}}{\lambda_m}$$

where λ_m is the wavelength at maximum intensity.

From graph, $\lambda_1 < \lambda_2 < \lambda_3$

i.e. λ_3 is the highest and λ_1 is the lowest.

$\because$ λ_3 corresponds with T_2 on graph,

$\therefore$ T_2 is the lowest temperature

Similarly, λ_1 corresponds with temperature T_1

$\therefore$ T_1 is the highest temperature, because λ_1 is the lowest

$\therefore$ $T_1 > T_3 > T_2$.

Illustration 27

A body takes 10 min to cool from 60°C to 50°C. Temperature of surroundings is 25°C. Find the temperature of body after next 10 min.

Soln.: According to principle of rate of cooling,

$$\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1+\theta_2}{2} - \theta_0\right]$$

where θ_0 = room temperature

$$\therefore \quad \frac{60-50}{10} = K\left[\frac{60+50}{2} - 25\right] = K \times 30$$

$$K = \frac{1}{30} \quad \text{...(i)}$$

After another 10 min, let the temperature be θ.

$$\therefore \quad \frac{50-\theta}{10} = K\left[\frac{50+\theta}{2} - 25\right] \quad \text{...(ii)}$$

or $\frac{10}{50-\theta} = \frac{30 \times 2}{\theta} \Rightarrow \theta = 42.85°\text{C}$ (using (i))

$\therefore$ $\theta = 42.85°$C.

GREEN HOUSE EFFECT

- The earth's atmosphere is the transparent to visible radiations and infra-red radiations from sun. The earth absorbs all these radiations and releases longer wavelength infra-red radiations which can not pass through lower atmosphere, they get absorbed by atmosphere molecules leading to rise in the earth's temperature. This phenomenon is called green house effect. The green house effect keeps the earth's surface warm at night. However, an increased concentration of gases like CO_2 which absorbs infra-red radiations is causing global warming which is now a matter of international concern.