Illustration 23

Two slabs of thickness d_1 and d_2 and of conductivities K_1 and K_2 are placed in contact and combination behaves as a single slab of conductivity K given by

(a) $\frac{d_1}{K_1}$ (b) $d_1K_1 + d_2K_2 = K$

(c) $\frac{d_1}{K_1} + \frac{d_2}{K_2} = \frac{d_1 + d_2}{K}$ (d) $K(d_1 + d_2) = K_1K_2d_1d_2$

Soln. (c) : Thermal resistances behave analogous to electrical resistances.

In series,

$$R_{eq} = R_1 + R_2 \quad \text{Now, } R_{thermal} = \frac{\Delta x}{KA}$$

Here, $\frac{(d_1 + d_2)}{KA} = \frac{d_1}{K_1A} + \frac{d_2}{K_2A} \Rightarrow \frac{d_1}{K_1} + \frac{d_2}{K_2} = \frac{d_1 + d_2}{K}$.

$\Rightarrow$ (c) is correct.

Illustration 24

A sphere is at a temperature 600 K. Its cooling rate is R in an external environment of 200 K. Its temperature falls to 400 K, then cooling rate R' will be

(a) $\frac{3}{16}R$ (b) $\frac{16}{3}R$

(c) $\frac{9}{27}R$ (d) None of these

Soln. (a) : From Stefan's law, net rate of heat energy lost per second

$$R = \varepsilon\sigma A(T^4 - T_0^4)$$

where T is the temperature of the body and T_0 is the temperature of the surroundings.

Here $R = \varepsilon\sigma A(600^4 - 200^4)$

$$R' = \varepsilon\sigma A(400^4 - 200^4) \Rightarrow \frac{R'}{R} = \left[\frac{400^4 - 200^4}{600^4 - 200^4}\right]$$

$$R' = \left(\frac{4^4 - 2^4}{6^4 - 2^4}\right)R$$

Hence, $R' = \frac{3}{16}R$. $\Rightarrow$ (a) is correct.

Illustration 25

A black body has maximum wavelength λ_m at 2000 K. Its corresponding wavelength at 3000 K will be

(a) $\frac{3}{2}\lambda_m$ (b) $\frac{2}{3}\lambda_m$ (c) $\frac{16}{81}\lambda_m$ (d) $\frac{81}{16}\lambda_m$

Soln. (b) : Using Wien's displacement law,

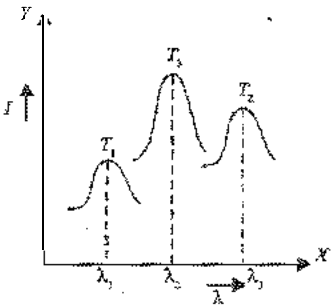
$$\lambda_{max} \cdot T = \text{constant}$$

Here $\lambda'_m \cdot (3000\text{ K}) = \lambda_m (2000\text{ K}) \Rightarrow \lambda'_m = \frac{2}{3}\lambda_m$.

$\Rightarrow$ (b) is correct.

Illustration 26

The plots of intensity versus wavelength for three black bodies at temperatures T_1, T_2 and T_3 respectively are shown. Which, of these temperatures are the lowest and the highest? Grade T_1, T_2 and T_3.



Soln.: According to Wien's law,

$$\lambda_m T = \text{constant} \Rightarrow T = \frac{\text{Constant}}{\lambda_m}.$$

where λ_m is the wavelength at maximum intensity.

From graph, $\lambda_1 < \lambda_2 < \lambda_3$

i.e. λ_3 is the highest and λ_1 is the lowest.

$\because$ λ_3 corresponds with T_2 on graph,

$\therefore$ T_2 is the lowest temperature

Similarly, λ_1 corresponds with temperature T_1

$\therefore$ T_1 is the highest temperature, because λ_1 is the lowest

$\therefore$ $T_1 > T_3 > T_2$.

Illustration 27

A body takes 10 min to cool from 60°C to 50°C. Temperature of surroundings is 25°C. Find the temperature of body after next 10 min.

Soln.: According to principle of rate of cooling,

$$\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

where θ_0 = room temperature

$$\therefore \quad \frac{60 - 50}{10} = K\left[\frac{60 + 50}{2} - 25\right] = K \times 30$$

$$K = \frac{1}{30} \quad \text{...(i)}$$

After another 10 min, let the temperature be θ.

$$\therefore \quad \frac{50 - \theta}{10} = K\left[\frac{50 + \theta}{2} - 25\right] \quad \text{...(ii)}$$

or $\frac{10}{50 - \theta} = \frac{30 \times 2}{\theta} \Rightarrow \theta = 42.85°C$ (using (i))

$\therefore$ $\theta = 42.85°C$.

GREEN HOUSE EFFECT

- The earth's atmosphere is the transparent to visible radiations and infra-red radiations from sun. The earth absorbs all these radiations and releases longer wavelength infra-red radiations which can not pass through lower atmosphere, they get absorbed by atmosphere molecules leading to rise in the earth's temperature. This phenomenon is called green house effect. The green house effect keeps the earth's surface warm at night. However, an increased concentration of gases like CO_2 which absorbs infra-red radiations is causing global warming which is now a matter of international concern.

CONCEPT MAP

Properties of Bulk Matter

SOLIDS

Elasticity : The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming forces.

Stress : Restoring force acting per unit area inside the body.

Strain : The ratio of change in configuration to the original configuration of the body.

Hooke's Law : Stress is directly proportional to strain within the elastic limit.

$\frac{\text{Stress}}{\text{Strain}} = E$ (modulus of elasticity)

Poisson's Ratio (σ) : It is the ratio of lateral strain to longitudinal strain.

Modulus of Elasticity : It is the ratio of stress to the corresponding strain, produced within the elastic limit.

Elastic Potential Energy :

$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$

$= \frac{1}{2} \times \text{Young's modulus} \times (\text{Strain})^2 \times \text{Volume}$

Young's Modulus : It is the ratio of normal stress to the longitudinal strain, within the elastic limit.

$Y = \frac{Fl}{a\Delta l}$

Bulk Modulus : It is the ratio of normal stress to the volumetric strain, within the elastic limit.

$B = -\frac{FV}{a\Delta V}$

Modulus of Rigidity : It is the ratio of tangential stress to the shearing strain, within the elastic limit.

$G = \frac{F}{a\theta}$

Excess pressure inside a liquid drop is given by,

$P = \frac{2S}{R}$

Excess pressure inside a soap bubble is give by,

$P = \frac{4S}{R}$

Excess pressure inside an air bubble in a liquid is given by,

$P = \frac{2S}{R}$

FLUIDS

At rest

Pressure : $P = \frac{\text{Thrust}}{\text{Area}} = \frac{F}{a} = h\rho g$

Surface Tension : $S = \frac{\text{Force}}{\text{Length}} = \frac{F}{l}$

Surface Energy : $S \times$ Area of liquid surface

Capillarity : The phenomenon of rise or fall of liquid in a capillary tube.

$h = \frac{2S\cos\theta}{r\rho g}$

Angle of Contact (θ) : The angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid.

At motion

Viscosity : It is the viscous drag of fluid.

Streamline Flow : Every particle of the liquid follows exactly the path of its preceding particle with same velocity.

Laminar Flow : Liquid moves in the form of layers.

Turbulent Flow : Liquid moves with a velocity greater than its critical velocity.

Bernoulli's Theorem :

$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Reynold's Number :

$N_R = \frac{\rho D v_c}{\eta}$

Stoke's Law : Backward dragging force acting on small spherical body moving in a fuild is given by

$F = 6\pi\eta r v$

Terminal Velocity :

$v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$

Critical Velocity : It is that velocity of liquid flow upto which the flow of liquid is streamlined and above which the flow becomes turbulent.

Symbols Used

- F = force applied
- a = area of cross-section
- l = initial length
- Δl = change in length
- V = initial volume
- ΔV = change in volume
- θ = shearing strain
- h = height/depth
- ρ = density
- g = acceleration due to gravity
- r = radius of tube
- R = radius of drop/bubble
- η = coefficient of viscosity
- D = diameter of tube
- $(\rho - \sigma)$ = difference in density of the body and liquid