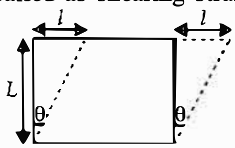
- Strains are of three types :
  - **Longitudinal strain** $= \dfrac{\text{Change in length}}{\text{Original length}} = \dfrac{\Delta L}{L}$
  - **Volumetric strain** $= \dfrac{\text{Change in volume}}{\text{Original volume}} = \dfrac{\Delta V}{V}$
- **Shearing strain :** When there is a change in the shape of a body without any change in its volume, the strain produced is called as shearing strain.



If a body is acted upon by an external force tangential to the surface of the body and the opposite surface is being kept fixed, then the angle through which the line perpendicular to the stationary surface is turned is called angle of a shear θ or shearing strain.

$$\text{Shearing strain} = \theta = \frac{l}{L}$$

HOOKE'S LAW

- According to Hooke's law, within the elastic limit, the stress applied to a body is directly proportional to the corresponding strain.
  Stress ∝ Strain
  or Stress $= E \times$ Strain or $\dfrac{\text{Stress}}{\text{Strain}} = E$
  Where E is the constant of proportionality and is known as coefficient of elasticity or modulus of elasticity.
- Hooke's law is an empirical law and is found to be valid for most materials. However, there are some materials which do not exhibit this linear relationship.

Illustration 1

A rope 1 cm in diameter breaks if the tension in it exceeds 500 N. The maximum tension that may be given to a similar rope of diameter 2 cm is

(a) 500 N (b) 250 N
(c) 1000 N (d) 2000 N

Soln. (d) : The fracture point for a given material happens at a constant stress.

$$\text{Breaking stress} = \frac{500\ \text{N}}{\pi(1\ \text{cm})^2} = \frac{T_{max}}{\pi(2\ \text{cm})^2}$$

⇒ $T_{max} = 500\ \text{N} \times 4 = 2000\ \text{N}$

⇒ (d) is correct.

Illustration 2

A metal wire can sustain the weight of 25 kg wt without breaking. The wire is cut into three equal parts. How much weight can each part sustain?

Soln.: Breaking stress $= \dfrac{\text{Maximum force applied}}{\text{Area of cross-section}}$

It does not depend on the length of wire. It depends on the material. Hence breaking stress remains same. Each part can sustain 25 kg wt.

Illustration 3

The breaking stress for a metal wire is $2 \times 10^8\ \text{N m}^{-2}$. Density of metal $= 8 \times 10^3\ \text{kg m}^{-3}$. Calculate the maximum length of copper wire which will not break under its own weight, when held in vertical position.

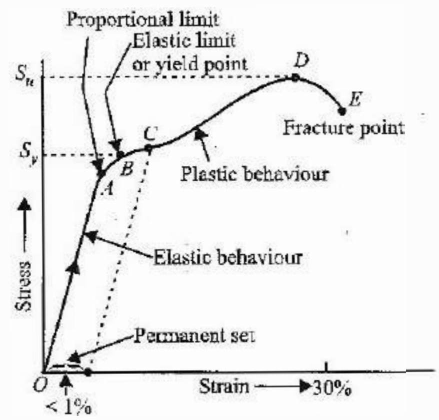
Soln.: Breaking stress $= \dfrac{Mg}{A} = \dfrac{(AL\rho)g}{A} = L\rho g$

$\therefore\ L = \dfrac{\text{Breaking stress}}{\rho g}$

or $L = \dfrac{2 \times 10^8}{8 \times 10^3 \times 10}$ or $L = 2500$ m.

STRESS-STRAIN CURVE

- The stress-strain curves vary from material to material. These curves help us to understand how a given material deforms with increasing loads. A typical stress-strain curve for a ductile metal is shown below.



- **Elastomers :** Substances like tissue of aorta, rubber etc., which can be stretched to cause large strains are called as elastomers.

MODULUS OF ELASTICITY

- It is defined as the ratio of stress to the corresponding strain produced within the elastic limit. Modulus of elasticity is of three types :
  - **Young's modulus,** $Y = \dfrac{\text{Normal stress}}{\text{Longitudinal strain}}$
    $$= \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L} = \frac{FL}{\pi r^2 \Delta L}$$
  - **Bulk modulus,** $B = \dfrac{\text{Normal stress}}{\text{Volumetric strain}}$
    $$= \frac{-F/A}{\Delta V/V} = -\frac{PV}{\Delta V}.$$
    –ve sign shows that with an increase in pressure, a decrease in volume occurs.
  - **Modulus of rigidity,** $G = \dfrac{\text{Tangential stress}}{\text{Shearing strain}}$
    $$= \frac{F/A}{\theta} = \frac{F}{A\theta}.$$
    Modulus of rigidity is also called as **shear modulus of rigidity.**

- The value of moduli of elasticity is independent of the magnitude of the stress and strain. It depends only on the nature of the material of the body.
- For a given material there can be different moduli of elasticity depending on the type of stress applied and the strain resulting.
- The moduli of elasticity have same dimensional formula and units as that of stress since strain is dimensionless, *i.e.*, the dimensional formula for Y, B and G is $[ML^{-1}T^{-2}]$ while the SI unit of Y, B and G is N m^{-2} or Pa and the CGS unit of Y, B and G is dyne cm^{-2}.
- Young's modulus is defined only for solids, but not for liquids and gases.
- Young's modulus of a perfect rigid body is infinity.
- Greater the value of Young's modulus of a material, larger is its elasticity. Therefore steel is more elastic than copper.
- The reciprocal of bulk modulus is known as **compressibility.**

  $$\text{Compressibility} = \frac{1}{B}$$

- The SI unit of compressibility is N^{-1} m^2 and CGS unit is $dyne^{-1}$ cm^2.
- Bulk modulus is relevant for solids, liquids and gases.
- Bulk modulus for gases are very low while that for liquids and solids are very high.

  $$B_{solid} > B_{liquid} > B_{gas}$$

- For gases, bulk modulus are of two types:
  - Isothermal bulk modulus, $B_{iso} = P$ (pressure exerted by the gas)
  - Adiabatic bulk modulus, $B_{ad} = \gamma P$

    where $\gamma = C_P/C_V$.

  $$\therefore \quad \frac{B_{ad}}{B_{iso}} = \gamma > 1; \quad B_{ad} > B_{iso}$$

  Therefore adiabatic bulk modulus is greater than isothermal bulk modulus.
- Shear modulus is defined for solids only. Shear modulus (or modulus of rigidity) for a solid is generally less than its Young's modulus. For most of the solid materials, the value of shear modulus is one third of the Young's modulus, *i.e.*, $G = Y/3$.

Illustration 4

A thick pipe of length 5 m is made of rubber having $Y = 10^6$ N m^{-2}, density = 1500 kg m^{-3}. It is held in vertical position when suspended from a ceiling. Calculate the extension produced in the pipe under its own weight.

Soln.: Extension, $l = \dfrac{MgL}{AY}$

Its weight is supposed to act at its centre of gravity which lies midway at $\dfrac{L}{2}$. The centre of gravity is displaced under its own weight of pipe.

$$\therefore \quad \text{Extension } l = \frac{(AL\rho)g}{AY} \times \left(\frac{L}{2}\right) = \frac{L^2\rho g}{2Y}$$

$$\text{or} \quad l = \frac{(5)^2 \times 1500 \times 10}{2 \times 10^6} = \frac{25 \times 15}{2 \times 1000}$$

$$l = 0.1875 \text{ m.}$$

Illustration 5

Two wires P and Q are made of same material. The wire P has a length l and diameter r while the wire Q has a length $2l$ and diameter $r/2$. If the two wires are stretched by the same force, the elongation in P divided by the elongation in Q is

(a) 1/8 (b) 1/4
(c) 4 (d) 8

Soln. (a) : $Y = \dfrac{F/A}{x/L}$

So, the elongation x is given as,

$$x = \left(\frac{F \cdot L}{A \cdot Y}\right) = \frac{F \cdot L}{\pi R^2 \cdot Y}$$

For wires P and Q are made of same material or in other words same Y, stretched by the same force.

$$\frac{x_P}{x_Q} = \left(\frac{L_P}{R_P^2}\right)\left(\frac{R_Q^2}{L_Q}\right) = \left(\frac{L_P}{L_Q}\right)\cdot\left(\frac{R_Q}{R_P}\right)^2$$

Here $L_P = l$, $R_P = \dfrac{r}{2}$, $L_Q = 2l$ and $R_Q = \dfrac{r}{4}$

$$\Rightarrow \quad \frac{x_P}{x_Q} = \left(\frac{l}{2l}\right)\cdot\left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

$\Rightarrow$ (a) is correct.

Illustration 6

The length of a metal wire is l_1 when the tension in it is T_1 and is l_2 when the tension is T_2. The natural length of the wire is

(a) $\dfrac{l_1 + l_2}{2}$ (b) $\sqrt{l_1 l_2}$

(c) $\dfrac{l_1T_2 - l_2T_1}{T_2 - T_1}$ (d) $\dfrac{l_1T_2 + l_2T_1}{T_2 + T_1}$

Soln. (c) : Let the length of the wire be l_0.
The elongation $x_1 = (l_1 - l_0)$ and the elongation $x_2 = (l_2 - l_0)$

As $Y = \dfrac{F \cdot L}{A \cdot x}$

For different tensions of the same wire,

$$\frac{F_1}{x_1} = \frac{F_2}{x_2} \Rightarrow \frac{T_1}{(l_1 - l_0)} = \frac{T_2}{(l_2 - l_0)}$$

$$T_1(l_2 - l_0) = T_2(l_1 - l_0)$$

$$T_1l_2 - T_1l_0 = T_2l_1 - T_2l_0$$

$$(T_2 - T_1)l_0 = T_2l_1 - T_1l_2 \Rightarrow l_0 = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}$$

$\Rightarrow$ (c) is correct.