Illustration 7

The bulk modulus of a metal is 8×10^9 N m^{-2}. Its density is 11 g cm^{-3}. What will be the density of metal under a pressure of 20,000 N cm^{-2}?

Soln.: Bulk modulus, $B = -\frac{PV}{\Delta V}$

$\therefore \quad \Delta V = -\frac{PV}{B} = -\frac{2 \times 10^4 \times 10^4 V}{8 \times 10^9} = -\frac{V}{40}$

$\therefore$ New volume $= V - \frac{V}{40} = \frac{39V}{40}$

Let the new density of metal = ρ'gram cm^{-3}.

$\therefore$ Mass of metal $= \frac{39V}{40} \times \rho'$

or $V \times 11 = \frac{39V \times \rho'}{40}$ or $\rho' = \frac{440}{39} \frac{\text{gram}}{\text{cm}^3}$.

POISSON'S RATIO (σ)

- Poisson's ratio, $\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-\Delta r / r}{\Delta L / L}$

  –ve sign shows that if the length increases, then the radius of the wire decreases.
- σ has no units and dimensions.
- Theoretically, σ lies between –1 and $+\frac{1}{2}$.
- Practically σ lies between zero and $+\frac{1}{2}$.

RELATION BETWEEN *Y, B, G* AND σ

- $Y = 3B(1 - 2\sigma)$
- $Y = 2G(1 + \sigma)$
- $\sigma = \frac{3B - 2G}{2G + 6B}$
- $\frac{9}{Y} = \frac{1}{B} + \frac{3}{G}$

Elastic Potential Energy Stored in a Stretched Wire and Breaking Force

- Elastic potential energy stored in a stretched wire = work done in stretching the wire = $\frac{1}{2}$ × stretching force × extension in length of wire

  $$U = \frac{1}{2} F \times \Delta L = \frac{1}{2} \frac{F}{A} \times \frac{\Delta L}{L} \times AL$$

  $$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$
- Elastic potential energy stored per unit volume of a stretched wire

  $$u = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$$

  $$= \frac{1}{2} \times \frac{1}{Y} \times (\text{stress})^2$$
- **Breaking force** = Breaking stress × Area of cross section of the wire.

Illustration 8

The area of cross-section of railway track is 0.01 m^2. The temperature variation is 10°C. Coefficient of linear expansion of steel = 10^{-5}/°C.

(Young's modulus of steel = 10^{11} Nm^{-2})

Calculate the energy stored per meter in the track.

Soln.: Let α = coefficient of linear expansion.

$\therefore$ Elastic energy $= \frac{1}{2} \times$ stress × strain × volume

or $U = \frac{1}{2} \times (Y \times \text{strain}) \times \text{strain} \times \text{volume}$

or $U = \frac{1 \times Y \times (\text{strain})^2 \times \text{volume}}{2}$

$\alpha = \frac{l}{L \times t} = \frac{\text{strain}}{t}$

$\because \quad \alpha = \frac{\text{change in length}}{\text{original length} \times \text{temperature change}}$

or strain = αt

$\therefore \quad U = \frac{Y \times \alpha^2 t^2 \times (\text{area} \times \text{length})}{2}$

or $U = \frac{10^{11} \times (10^{-5})^2 \times (10)^2 \times 0.01 \times 1}{2} = 5$ J

$\therefore$ Energy stored = 5 J.

SOME IMPORTANT FACTS ABOUT ELASTICITY

- Young's modulus is numerically equal to the normal stress which will double the length of a wire.
- **Elongation in a wire by its own weight :** If a wire of length L and cross-sectional area A is stretched by a force F, then by the definition of Y,

  $$\Delta L = \frac{FL}{AY} \qquad \left[\text{As } Y = \frac{FL}{A\Delta L}\right]$$

  In case of elongation by its own weight, ($F = Mg$) will act at centre of gravity of the wire, so that length of wire which is stretched is (L/2).

  $$\therefore \quad \Delta L = \frac{Mg(L/2)}{AY} = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y} \quad [\text{As } M = \rho AL]$$
- **Thermal stress :** If a rod is fixed between two rigid supports, due to change in temperature its length will change and so it will exert a normal stress on the supports. This stress is called as thermal stress.

  Thermal stress = $Y\alpha\Delta T$

  where α is the coefficient of linear expansion and ΔT is the change in temperature.
- **Interatomic force constant (k)**

  $$k = \frac{\text{Interatomic force}}{\text{Change in interatomic distance}} = \frac{F_0}{\Delta r}$$
- If the Young's modulus for a material is Y and the equilibrium distance between the atom is r_0, then $k_0 = Yr_0$

- In case of twisting of a cylinder (or wire) of length L and radius r, elastic restoring couple per unit twist is given by

$$C = \frac{\pi G r^4}{2L}$$

where G is modulus of rigidity of the material of wire.

- Depression of a beam loaded at the middle by a load W and supported at the ends is

$$\delta = \frac{WL^3}{48YI_g}$$

where L is the length of a beam, Y is the Young's modulus for the material of the beam, and I_g is the geometrical moment of inertia.

For a beam of circular cross-section of radius r,

$$I_g = \frac{\pi r^2}{4}$$

For a beam of rectangular cross-section of breadth b and thickness d, $I_g = \frac{bd^3}{12}$

FLUIDS

- Those substances which can flow are called as fluids. Fluids include both liquids and gases.
- We study about fluids at rest in hydrostatics and about fluids in motion in hydrodynamics.

DENSITY

- Density of a substance is defined as the mass per unit volume of the substance.

$$\text{Density, } \rho = \frac{\text{Mass }(M)}{\text{Volume }(V)}$$

- Density is a positive scalar quantity. Its dimensional formula is $[ML^{-3}T^0]$. The SI unit of density is kg m^{-3} and CGS unit is g cm^{-3}.

  1 g cm^{-3} = 10^3 kgm^{-3}

- Density of substance means the ratio of mass of the substance to the volume occupied by the substance while density of a body means the ratio of mass of a body to the volume of the body. For a solid body,

  Density of body = Density of substance

  For a hollow body, density of body is lesser than that of substance [As $V_{body} > V_{substance}$].

- When immiscible liquids of different densities are poured in a container, the liquid of highest density will be at the bottom while that of lowest density at the top and interfaces will be plane.
- The maximum density of water at 4°C (277 K) which is 1.0×10^3 kg m^{-3}.
- A liquid is largely incompressible and its density is therefore, nearly constant at all pressures. Gases, on the other hand, exhibit a large variation in densities with pressure.
- **Relative density** : Relative density of a substance is defined as the ratio of its density to the density of water at 4°C.

$$\text{Relative density} = \frac{\text{Density of a substance}}{\text{Density of water at 4°C}}$$

- Relative density is a positive scalar quantity. It has no units and dimensions.
- Relative density is also known as **specific gravity.**
- The value of relative density of a substance is same in both CGS and SI system.
- If two liquids of masses m_1, m_2 and densities ρ_1, ρ_2 are mixed together, then the density of the mixture is given by

$$\rho = \frac{m_1 + m_2}{(m_1/\rho_1) + (m_2/\rho_2)}$$

If $m_1 = m_2 = m$, $\rho = \frac{m + m}{(m/\rho_1) + (m/\rho_2)} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$

- If two liquids of volume V_1 and V_2 of density ρ_1 and ρ_2 are mixed together then density of the mixture,

$$\rho = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2} \quad \text{If } V_1 = V_2 = V, \ \rho = \frac{(\rho_1 + \rho_2)}{2}.$$

PRESSURE

- **Thrust** : The total normal force exerted by liquid at rest on a given surface in contact with it is called thrust of liquid on that surface.
- **Pressure** : It is defined as the thrust acting per unit area of the surface in contact with liquid.

$$P = \frac{\text{Thrust }(F)}{\text{Area }(A)} = \frac{F}{A}$$

- Pressure is a scalar quantity. Its dimensional formula is $[ML^{-1}T^{-2}]$.
- The SI unit of pressure is N m^{-2}. It has been named as pascal (Pa) in the honour of French scientist Blaise Pascal.
- Other common units of pressure are :
  - 1 atm = 1.01×10^5 Pa
  - 1 bar = 10^5 Pa
  - 1 torr = 133 Pa
  - 1 mmofHg = 1 torr = 133 Pa.
- **Atmospheric pressure** : The pressure exerted by atmosphere is called atmospheric pressure. At S.T.P. the value of atmospheric pressure is 1.01×10^5 N m^{-2} or 1.01×10^6 dyne cm^{-2}.

Illustration 9

10 g of a liquid of density 5 g cm^{-3} is mixed with 12 g of another immiscible liquid of density 4 g cm^{-3}. Find the density of mixture.

Soln.: Density of mixture = $\frac{[\text{Total mass of mixture}]}{[\text{Total volume of mixture}]}$

or Density of mixture, $D = \frac{m_1 + m_2}{\frac{m_1}{\sigma_1} + \frac{m_2}{\sigma_2}} = \frac{\sigma_1\sigma_2(m_1 + m_2)}{m_1\sigma_2 + m_2\sigma_1}$

$$\therefore D = \frac{5 \times 4(10 + 12)}{10 \times 4 + 12 \times 5} = \frac{20 \times 22}{40 + 60}$$

$$= \frac{20 \times 22}{100} = 4.4 \text{ g cm}^{-3}.$$

$\therefore$ Density of mixture of liquids = 4.4 g cm^{-3}.