- In case of twisting of a cylinder (or wire) of length L and radius r, elastic restoring couple per unit twist is given by

$$C = \frac{\pi G r^4}{2L}$$

where G is modulus of rigidity of the material of wire.

- Depression of a beam loaded at the middle by a load W and supported at the ends is

$$\delta = \frac{WL^3}{48YI_g}$$

where L is the length of a beam, Y is the Young's modulus for the material of the beam, and I_g is the geometrical moment of inertia.
For a beam of circular cross-section of radius r,

$$I_g = \frac{\pi r^2}{4}$$

For a beam of rectangular cross-section of breadth b and thickness d, $I_g = \frac{bd^3}{12}$

FLUIDS

- Those substances which can flow are called as fluids. Fluids include both liquids and gases.
- We study about fluids at rest in hydrostatics and about fluids in motion in hydrodynamics.

DENSITY

- Density of a substance is defined as the mass per unit volume of the substance.

$$\text{Density, } \rho = \frac{\text{Mass } (M)}{\text{Volume } (V)}$$

- Density is a positive scalar quantity. Its dimensional formula is $[ML^{-3}T^0]$. The SI unit of density is kg m^{-3} and CGS unit is g cm^{-3}.
  1 g $cm^{-3} = 10^3$ kgm^{-3}
- Density of substance means the ratio of mass of the substance to the volume occupied by the substance while density of a body means the ratio of mass of a body to the volume of the body. For a solid body,
  Density of body = Density of substance
  For a hollow body, density of body is lesser than that of substance [As $V_{body} > V_{substance}$].
- When immiscible liquids of different densities are poured in a container, the liquid of highest density will be at the bottom while that of lowest density at the top and interfaces will be plane.
- The maximum density of water at 4°C (277 K) which is 1.0×10^3 kg m^{-3}.
- A liquid is largely incompressible and its density is therefore, nearly constant at all pressures. Gases, on the other hand, exhibit a large variation in densities with pressure.
- **Relative density** : Relative density of a substance is defined as the ratio of its density to the density of water at 4°C.

$$\text{Relative density} = \frac{\text{Density of a substance}}{\text{Density of water at 4°C}}$$

- Relative density is a positive scalar quantity. It has no units and dimensions.
- Relative density is also known as **specific gravity.**
- The value of relative density of a substance is same in both CGS and SI system.
- If two liquids of masses m_1, m_2 and densities ρ_1, ρ_2 are mixed together, then the density of the mixture is given by

$$\rho = \frac{m_1 + m_2}{(m_1/\rho_1) + (m_2/\rho_2)}$$

If $m_1 = m_2 = m$, $\rho = \frac{m+m}{(m/\rho_1)+(m/\rho_2)} = \frac{2\rho_1\rho_2}{\rho_1+\rho_2}$

- If two liquids of volume V_1 and V_2 of density ρ_1 and ρ_2 are mixed together then density of the mixture,

$$\rho = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2} \quad \text{If } V_1 = V_2 = V, \ \rho = \frac{(\rho_1+\rho_2)}{2}.$$

PRESSURE

- **Thrust** : The total normal force exerted by liquid at rest on a given surface in contact with it is called thrust of liquid on that surface.
- **Pressure** : It is defined as the thrust acting per unit area of the surface in contact with liquid.

$$P = \frac{\text{Thrust } (F)}{\text{Area } (A)} = \frac{F}{A}$$

- Pressure is a scalar quantity. Its dimensional formula is $[ML^{-1}T^{-2}]$.
- The SI unit of pressure is N m^{-2}. It has been named as pascal (Pa) in the honour of French scientist Blaise Pascal.
- Other common units of pressure are :
  - 1 atm = 1.01×10^5 Pa
  - 1 bar = 10^5 Pa
  - 1 torr = 133 Pa
  - 1 mm of Hg = 1 torr = 133 Pa.
- **Atmospheric pressure** : The pressure exerted by atmosphere is called atmospheric pressure. At S.T.P. the value of atmospheric pressure is 1.01×10^5 N m^{-2} or 1.01×10^6 dyne cm^{-2}.

Illustration 9

10 g of a liquid of density 5 g cm^{-3} is mixed with 12 g of another immiscible liquid of density 4 g cm^{-3}. Find the density of mixture.

Soln.: Density of mixture = $\frac{[\text{Total mass of mixture}]}{[\text{Total volume of mixture}]}$

or Density of mixture, $D = \frac{m_1 + m_2}{\frac{m_1}{\sigma_1} + \frac{m_2}{\sigma_2}} = \frac{\sigma_1\sigma_2(m_1+m_2)}{m_1\sigma_2 + m_2\sigma_1}$

$$\therefore D = \frac{5 \times 4(10+12)}{10 \times 4 + 12 \times 5} = \frac{20 \times 22}{40 + 60}$$

$$= \frac{20 \times 22}{100} = 4.4 \text{ g cm}^{-3}.$$

$\therefore$ Density of mixture of liquids = 4.4 g cm^{-3}.

BUOYANT FORCE OR BUOYANCY

- It is an upward force acting on the body immersed in a liquid.
- It is equal to the weight of liquid displaced by the immersed part of the body.
- The buoyant force acts at the **centre of buoyancy** which is the centre of gravity of the liquid displaced by the body when immersed in the liquid.
- The line joining the centre of gravity and centre of buoyancy is called **central line.**
- **Metacentre :** It is a point where the vertical line passing through the centre of buoyancy intersects the central line.

ARCHIMEDES' PRINCIPLE

- It states that when a body is immersed wholly or partly in a liquid at rest, it loses some of its weight. The loss in weight of the body in the liquid is equal to the weight of the liquid displaced by the immersed part of the body.

Application of Archimedes' Principle

- Determination of relative density of a body :

$$\text{Relative density} = \frac{\text{Weight of the body in air}}{\text{Loss in weight in water}}$$

$$= \frac{W_{air}}{W_{air} - W_{water}}$$

Determination of relative density of liquid :

$$\text{Relative density} = \frac{\text{Loss in weight in liquid}}{\text{Loss in weight in water}}$$

$$= \frac{W_{air} - W_{liquid}}{W_{air} - W_{water}}$$

Floatation

- When a body of density ρ and volume V is completely immersed in a liquid of density σ, following two forces act on the body :
  - Weight of body, $W = V\rho g$ acting vertically downwards through the centre of gravity.
  - Buoyant force or upward thrust, $w = V\sigma g$ equal to weight of the liquid displaced, acting vertically upwards through the centre of buoyancy.
- Depending upon the relative magnitudes of above two forces, following three cases are possible:
  - If $W > w$, the body will sink to the bottom of the liquid. It will be so when the density of solid by (ρ) is greater than the density of liquid (σ) *i.e.* $\rho > \sigma$.
  - If $W < w$, the body will rise above the surface of the liquid to such an extent that the weight of the liquid displaced by immersed part of the body (*i.e.* upward thrust) becomes equal to the weight of the body. Then the body will float. In this case the density of solid body is less than the density of liquid *i.e.* $\rho < \sigma$.
  - If $W = w$, the body is at rest any where in the liquid. The body will float if its whole volume is just immersed in the liquid. In this case the density of body is equal to density of liquid *i.e.* $\rho = \sigma$.
- The law of floatation states that a body will float, if the weight of the liquid displaced by the immersed part of the body is atleast equal to or greater than the weight of the body.

Illustration 10

When floated in water ($\sigma = 1000$ kg m^{-3}), only 0.6 fraction of volume of a solid is submerged. Find the density of liquid in which the solid just floats?

Soln.: When a solid just floats in a liquid, then density of solid = density of liquid.

Let the density of solid = ρ = density of liquid.

Weight of floating body = Weight of water displaced

$$V \times \rho \times g = (0.6V)\,\sigma\, g$$

or $\rho = 0.6\sigma = 0.6 \times 1000 = 600$ kg/m^3

Illustration 11

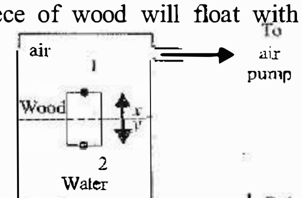
A piece of wood is floating in water kept in a bottle. The bottle is connected to an air pump. Neglect the compressibility of water. When more air is pushed into the bottle from the pump, the piece of wood will float with

(a) larger part in the water

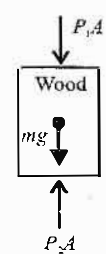
(b) lesser part in the water

(c) same part in the water

(d) it will sink



Soln. (c) : For equilibrium of wood,



$$P_2A = P_1A + mg$$

or $$P_2 = P_1 + \frac{mg}{A}$$

Now, $P_2 = P_{atm} + \rho g y$, $P_1 = P_{atm}$

$$\Rightarrow \quad (P_{atm} + \rho g y) = P_{atm} + \frac{mg}{A}$$

$$\frac{mg}{A} = \rho g y.$$

Thus y is independent of the P_{atm} or air pressure.

VISCOSITY

- It is the property of a fluid (liquid or gas) by virtue of which an internal resistance comes into play when the fluid is in motion and opposes the relative motion of its different layers.
- According to Newton, viscous force (F) of a liquid between two layers is given by

$$F = -\eta A \frac{dv}{dx}$$

where η = coefficient of viscosity of liquid

A = area of each layer, dv/dx = velocity gradient

–ve sign shows viscous force is acting in a direction opposite to the flow of a liquid.