- **Coefficient of viscosity :** It is defined as the ratio of shearing stress to the strain rate.

$$\eta = \frac{\text{Shearing stress}}{\text{Strain rate}} = \frac{F/A}{v/l} = \frac{Fl}{vA}$$

- The dimensional formula of η is $[ML^{-1}T^{-1}]$.
- The SI unit of η is poiseuille (Pl) or Pa s or N m^{-2} s.
- The CGS unit of η is dyne cm^{-2} s called poise.
  1 Pl = 10 poise
- The value of viscosity for ideal fluid is zero.
- Viscosity is due to transport of momentum.
- **Relative viscosity of liquid**

$$= \frac{\text{Viscosity of liquid}}{\text{Viscosity of water}} = \frac{\eta}{\eta_{water}}$$

Fluidity

- The reciprocal of viscosity is called as fluidity.

Poiseuille's Equation

- The rate of steady flow of a liquid *i.e.* volume of the liquid flowing per second (Q) at a steady rate through a horizontal capillary tube of length l, radius r, under a pressure difference P at its two ends is given by

$$Q = \frac{\pi P r^4}{8\eta l} = \frac{P}{R}$$

  This is known as Poiseuille's equation
  where η = coefficient of viscosity of the liquid
  $R = \frac{8\eta l}{\pi r^4}$ is called liquid resistance.
- A liquid flow through a capillary tube is analogous to the flow of electric current through a conductor. The rate of liquid flow Q corresponding to current or rate of flow of charge, the pressure difference P across the two ends of the capillary corresponds to potential difference across the two ends of the conductor and liquid resistance $\left(\frac{8\eta l}{\pi r^4}\right)$ correspond to electrical resistance.
- When two capillaries of lengths l_1 and l_2 and radii r_1 and r_2 respectively are joined in series through which the liquid is flowing, then the effective liquid resistance R_s is

$$R_s = R_1 + R_2 = \frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}$$

- When two capillaries of lengths l_1 and l_2 and radii r_1 and r_2 respectively are joined in parallel, through which liquid is flowing, then the effective resistance R_p is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \left(\frac{\pi r_1^4}{8\eta l_1} + \frac{\pi r_2^4}{8\eta l_2}\right)$$

Stoke's Law

- It states that the backward dragging force F acting on a small spherical body of radius r moving with velocity v through a fluid of viscosity η is given by
  $F = 6\pi\eta rv$.

Terminal Velocity

- When a spherical body falls through a viscous medium, its velocity increases, till the viscous drag plus upward thrust become equal to the weight of the body. After that the body moves with a constant velocity, called terminal velocity. Terminal velocity of a spherical body of radius r, density ρ while falling freely in a viscous medium of viscosity η and density σ is given by

$$v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

- The terminal velocity varies directly with the square of the radius of the body and inversely with the coefficient of viscosity of the medium. It also depends upon the densities of the body and the medium.

Variation of Viscosity

- **Variation of viscosity with temperature :**
  - Viscosity of a liquid decreases with rise in temperature. Viscosity of a liquid falls so rapidly with temperature that it becomes meaningless to mention the viscosity of a liquid without specifying its temperature. *e.g.* viscosity of water at 10°C has one-third its value at 80°C and that of castor oil falls even more rapidly.
  - Viscosity of a gas increases with rise in temperature as $\eta \propto \sqrt{T}$.
- **Variation of viscosity with pressure :**
  - With increase in pressure, the viscosity of liquid increases but the viscosity of water decreases.
  - Viscosities of gases are practically independent of pressure.

STREAMLINE FLOW, LAMINAR AND TURBULENT FLOW

- **Streamline flow :** It is that flow of liquid, in which every particle of the liquid follows exactly the path of its preceding particle and has the same velocity in magnitude and direction as that of its preceding particle while crossing through that point. A streamline flow is accompanied by streamlines of liquid.
- A streamline is the actual path followed by the procession of particles in a steady flow, which may be straight or curved such that tangent to it at any point indicates the direction of flow of liquid at that point.
- In streamline flow, two streamlines never cross each other.
- In a streamline flow, the velocity of liquid flow is less than the critical velocity of the liquid.
- A group of streamlines is called a **tube of flow.**
- **Laminar flow :** It is that steady flow in which the liquid moves in the form of layers. In this flow, one layer slides over the other layer of liquid. The velocity of liquid flow is always less than the critical velocity of the liquid. In general, laminar flow is a streamline flow.