- **Turbulent flow :** It is that flow of liquid, in which a liquid moves with a velocity greater than its critical velocity. The motion of the particles of liquid becomes disorder or irregular.
- In this flow, every particle of the liquid does not follow the path of its preceding particle and has not the same velocity in magnitude and direction as that of the preceding particle, while crossing a given point.
- In turbulent flow, the two streamlines cross each other.

CRITICAL VELOCITY

- It is that velocity of liquid flow, upto which the flow of liquid is streamlined and above which its flow becomes turbulent. Critical velocity of a liquid (v_c) flowing through a tube is given by

$$v_c = \frac{K\eta}{\rho r}$$

where ρ is the density of liquid flowing through a tube of radius r and η is the coefficient of viscosity of liquid.

REYNOLDS NUMBER

- It is a pure number which determines the nature of flow of liquid through a pipe. According to Reynold, the critical velocity v_c of a liquid flowing through a tube of diameter D is given by

$$v_c = \frac{N_R\eta}{\rho D} \quad \text{or} \quad N_R = \frac{\rho D v_c}{\eta}$$

where η is the coefficient of viscosity of the liquid, ρ is the density of liquid and N_R is a constant called Reynolds number.
- If the value of Reynolds number lies between 0 to 2000, the flow of liquid is streamlined or laminar. For values of N_R above 3000, the flow of liquid is turbulent and for values of N_R between 2000 to 3000, the flow of liquid is unstable changing from streamline to turbulent.

Illustration 12

Eight identical drops of water are falling through air with a steady velocity of 10 cm s^{-1}. All the drops merge into one bigger drop. Calculate the terminal velocity of bigger drop.

Soln.: Let R = radius of bigger drop

r = radius of each small drop

Equate volumes when the 8 drops merge into one.

$$\therefore \quad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

$$\therefore \quad R = 2r \qquad \ldots(i)$$

Since terminal velocity $V = \dfrac{2}{9}\dfrac{(\rho - \sigma)r^2 g}{\eta}$

$$\therefore \quad \frac{\text{Terminal velocity of bigger drop}}{\text{Terminal velocity of smaller drop}} = \frac{R^2}{r^2}$$

or $\dfrac{V}{10} = \left(\dfrac{2r}{r}\right)^2 = 4$ or $V = 40$ cm s^{-1}.

Illustration 13

Three capillaries of internal radii $2r$, $3r$ and $4r$, all of the same length, are joined end to end. A liquid passes through the combination and the pressure difference across this combination is 20.2 cm of mercury. The pressure difference across the capillary of internal radius $2r$ is

(a) 2 cm of Hg (b) 4 cm of Hg
(c) 8 cm of Hg (d) 16 cm of Hg

Soln. (d) : $V = \dfrac{\pi P r^4}{8\eta l}$

As capillaries of same length are joined end to end, the volume rate V across each of the capillaries is same.

Here, $P_1 \cdot r_1^4 = P_2 \cdot r_2^4 = P_3 \cdot r_3^4$

or $P_1(2r)^4 = P_2(3r)^4 = P_3(4r)^4 \Rightarrow 16P_1 = 81P_2 = 256P_3$

$$P_1 = \frac{81}{16}P_2 = 16P_3 \Rightarrow P_2 = \frac{16}{81}P_1 \text{ and } P_3 = \frac{P_1}{16}$$

It is given that:

$$(P_1 + P_2 + P_3) = 20.2 \text{ cm of Hg}$$

$$\Rightarrow \quad P_1\left(1 + \frac{16}{81} + \frac{1}{16}\right) = 20.2$$

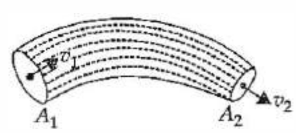
$$\text{or} \quad P_1\left(\frac{(81 \times 16) + (16 \times 16) + 81}{81 \times 16}\right) = 20.2$$

$$P_1 = 20.2\left[\frac{1296}{1296 + 256 + 81}\right] \approx 16 \text{ cm of Hg.}$$

$\Rightarrow$ (d) is correct.

EQUATION OF CONTINUITY

- If an incompressible and non-viscous liquid flowing through a tube of non-uniform cross-section with steady flow, the product of the area of cross-section and the velocity of flow remains same at every point in the tube.



$A_1 v_1 = A_2 v_2$

Av = constant.

This equation is known as continuity equation.
- In the steady flow of an incompressible liquid mass of the liquid flowing per second through a cross-section of the tube is constant.

i.e. $Av\rho$ = constant.
- In a streamline flow, the volume of the liquid flowing per second through a cross-section of the tube = Av where A is the area of cross-section and v is the velocity of the liquid flowing over that cross-section.
- The equation of continuity represents the conservation of mass in case of moving fluids.