CAPACITANCE

1. (i) $q \propto V \quad \Rightarrow \quad q = CV$

q : Charge on positive plate of the capacitor

C : Capacitance of capacitor.

V : Potential difference between positive and negative plates.

(ii) Representation of capacitor : —| |— , —| (—

(iii) Energy stored in the capacitor : $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{QV}{2}$

(iv) Energy density = $\frac{1}{2}\varepsilon_0\varepsilon_r E^2 = \frac{1}{2}\varepsilon_0 K E^2$

ε_r = Relative permittivity of the medium.

K= ε_r : Dielectric Constant

For vacuum, energy density = $\frac{1}{2}\varepsilon_0 E^2$

(v) Types of Capacitors :

(a) Parallel plate capacitor

$$C = \frac{\varepsilon_0\varepsilon_r A}{d} = K\frac{\varepsilon_0 A}{d}$$

A : Area of plates

d : distance between the plates(<< size of plate)

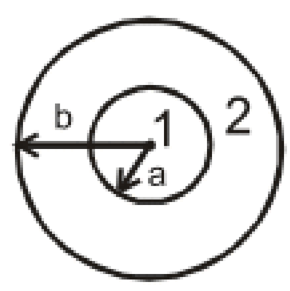
(b) Spherical Capacitor :

- Capacitance of an isolated spherical Conductor (hollow or solid)

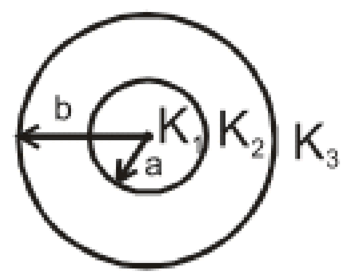
  $C = 4\pi\varepsilon_0\varepsilon_r R$

  R = Radius of the spherical conductor

- Capacitance of spherical capacitor



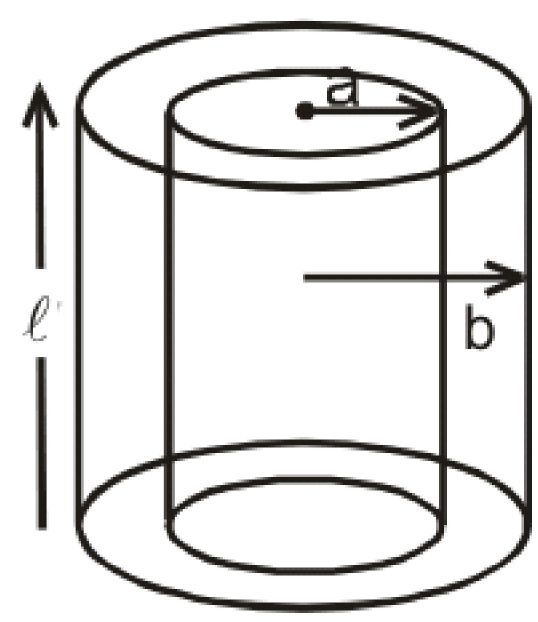
$$C = 4\pi\varepsilon_0\frac{ab}{(b-a)}$$



- $$C = \frac{4\pi\varepsilon_0 K_2 ab}{(b-a)}$$

(c) Cylindrical Capacitor : $\ell >> \{a,b\}$

Capacitance per unit length = $\frac{2\pi\varepsilon_0}{\ell n(b/a)}$ F/m



(vi) Capacitance of capacitor depends on

(a) Area of plates

(b) Distance between the plates

(c) Dielectric medium between the plates.

(vii) Electric field intensity between the plates of capacitor

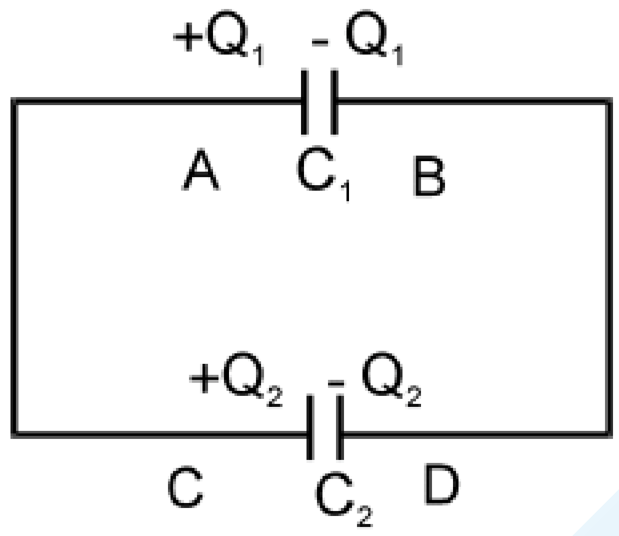
$$E = \frac{\sigma}{\varepsilon_0} = \frac{V}{d}$$

σ : Surface change density

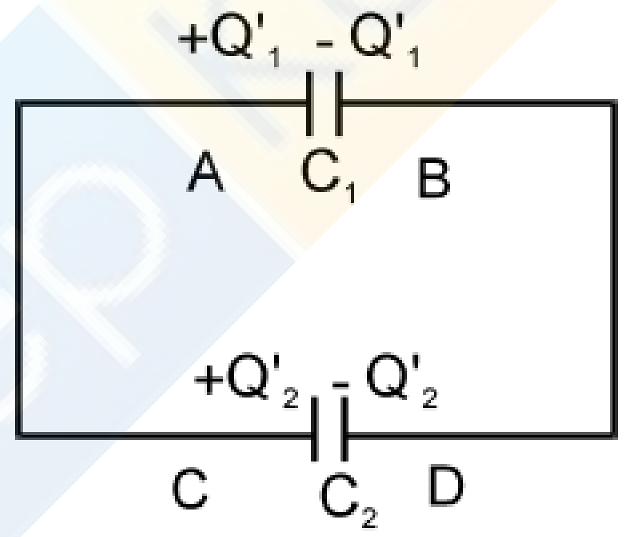
(viii) Force experienced by any plate of capacitor : $F = \frac{q^2}{2A\varepsilon_0}$

2. DISTRIBUTION OF CHARGES ON CONNECTING TWO CHARGED CAPACITORS:

When two capacitors are C_1 and C_2 are connected as shown in figure



Initially



Finally

(a) Common potential :

$$\Rightarrow \qquad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

(b) $Q_1' = C_1V = \frac{C_1}{C_1 + C_2}(Q_1 + Q_2)$

$$Q_2' = C_2 V = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$$

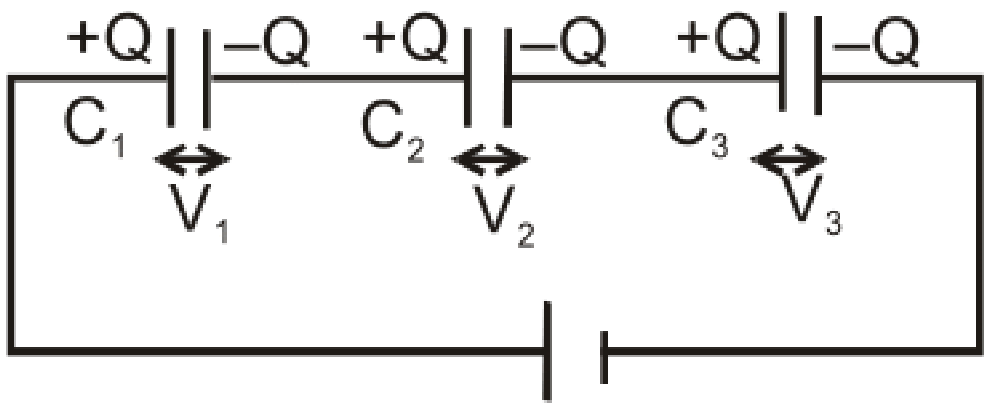
(c) Heat loss during redistribution :

$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

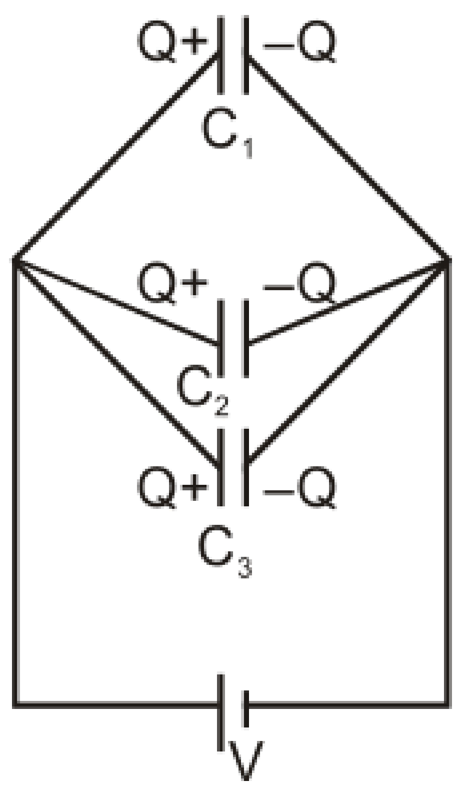
The loss of energy is in the form of Joule heating in the wire.

(i) Series Combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$



(ii) Parallel Combination :

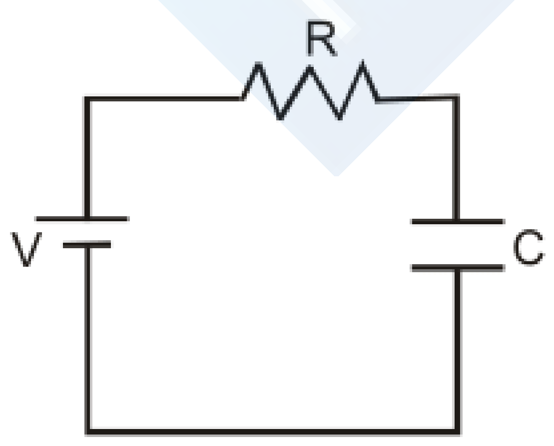


$$C_{eq} = C_1 + C_2 + C_3 \qquad Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$$

4. Charging and Discharging of a capacitor :

(i) Charging of Capacitor (Capacitor initially uncharged):

$$q = q_0 (1 - e^{-t/\tau})$$

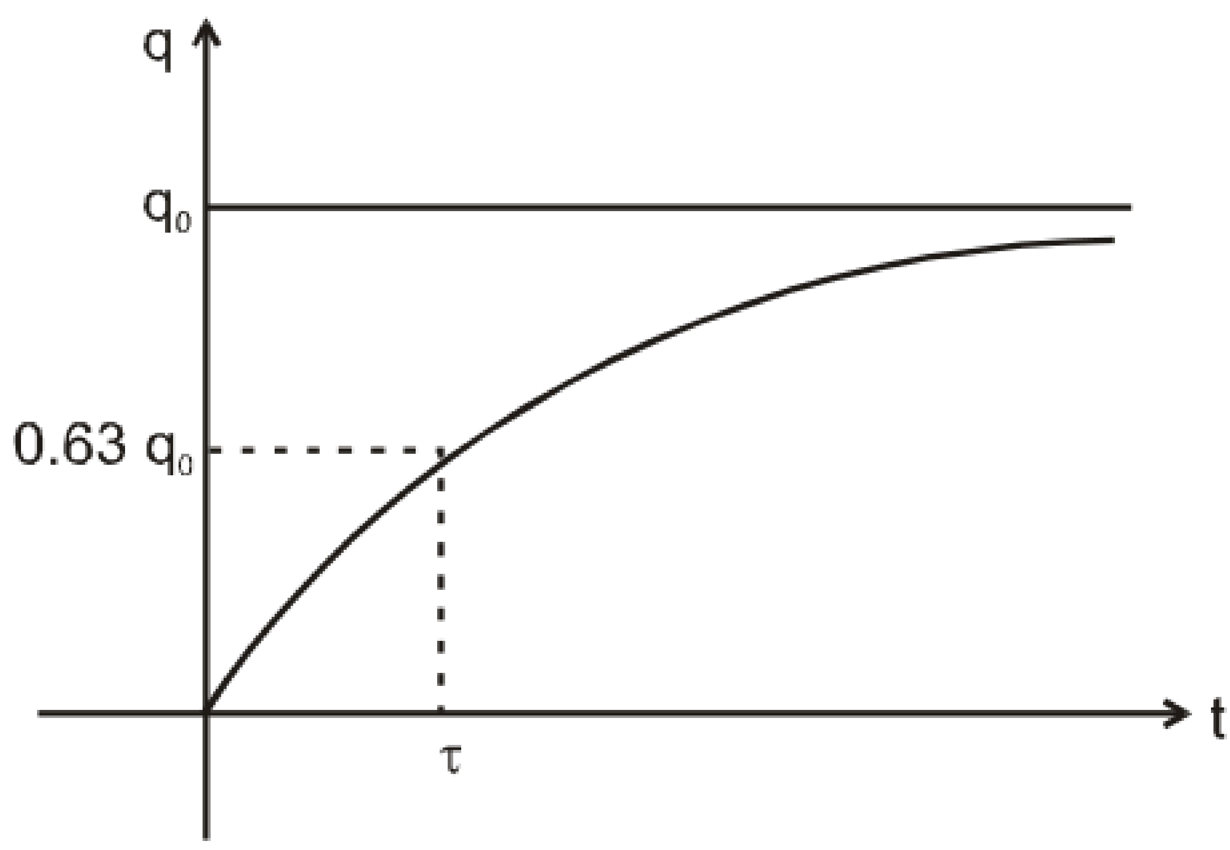


q_0 = Charge on the capacitor at steady state

q_0 = CV

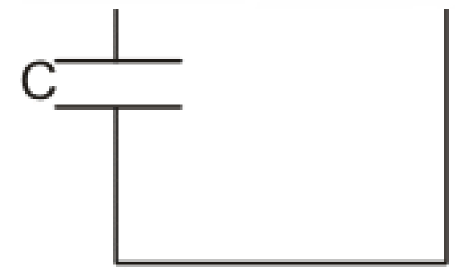
τ : Time constant $= CR_{eq.}$

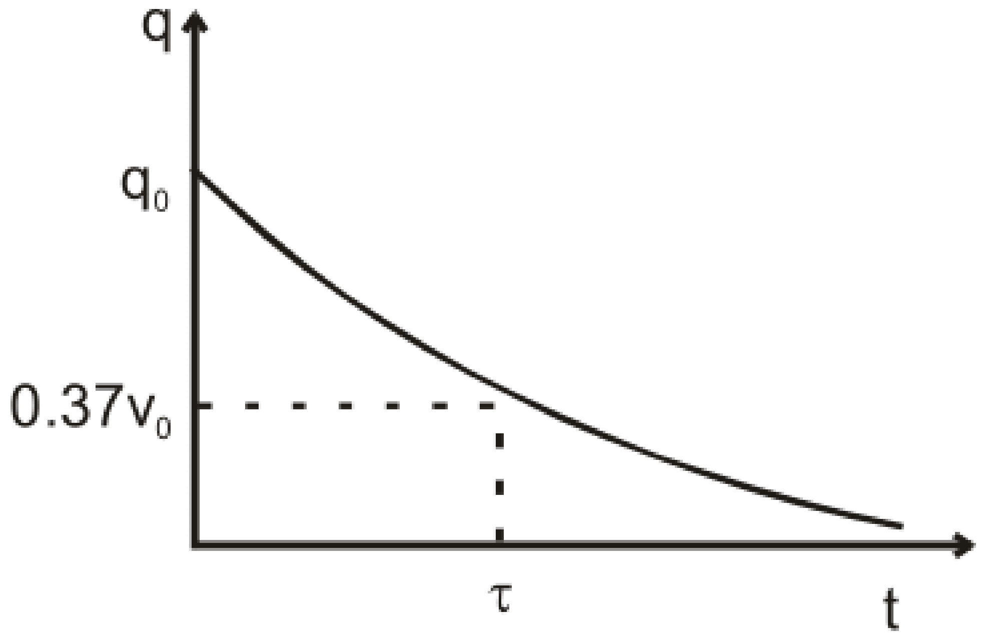
$$I = \frac{q_0}{\tau} e^{-t/\tau} = \frac{V}{R} e^{-t/\tau}$$



$q = q_0 e^{-t/\tau}$

q_0 = Initial charge on the capacitor

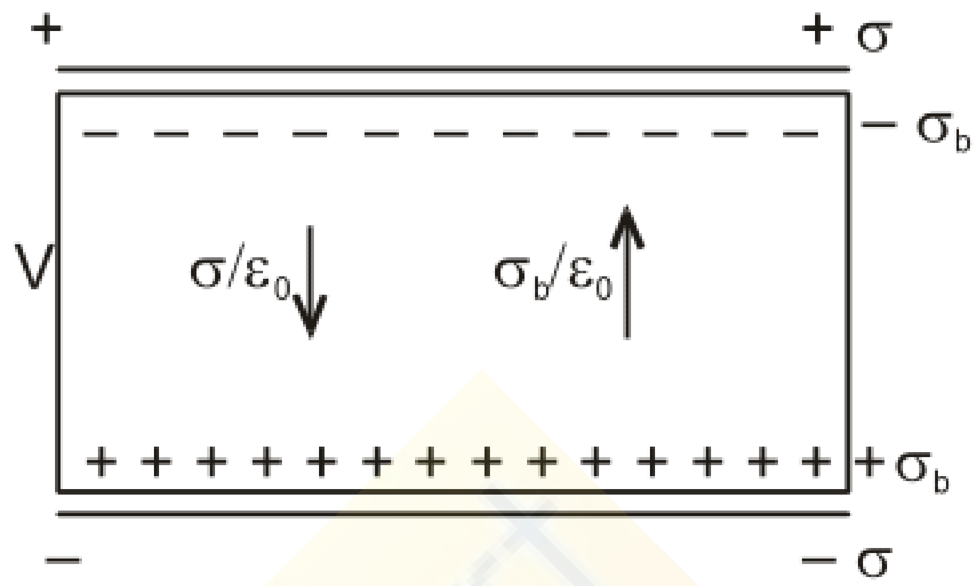




$$I = \frac{q_0}{\tau} e^{-t/\tau}$$

5. Capacitor with dielectric :

(i) Capacitance in the presence of dielectric :



$$C = \frac{K\varepsilon_0 A}{d} = KC_0$$

C_0 = Capacitance in the absence of dielectric.

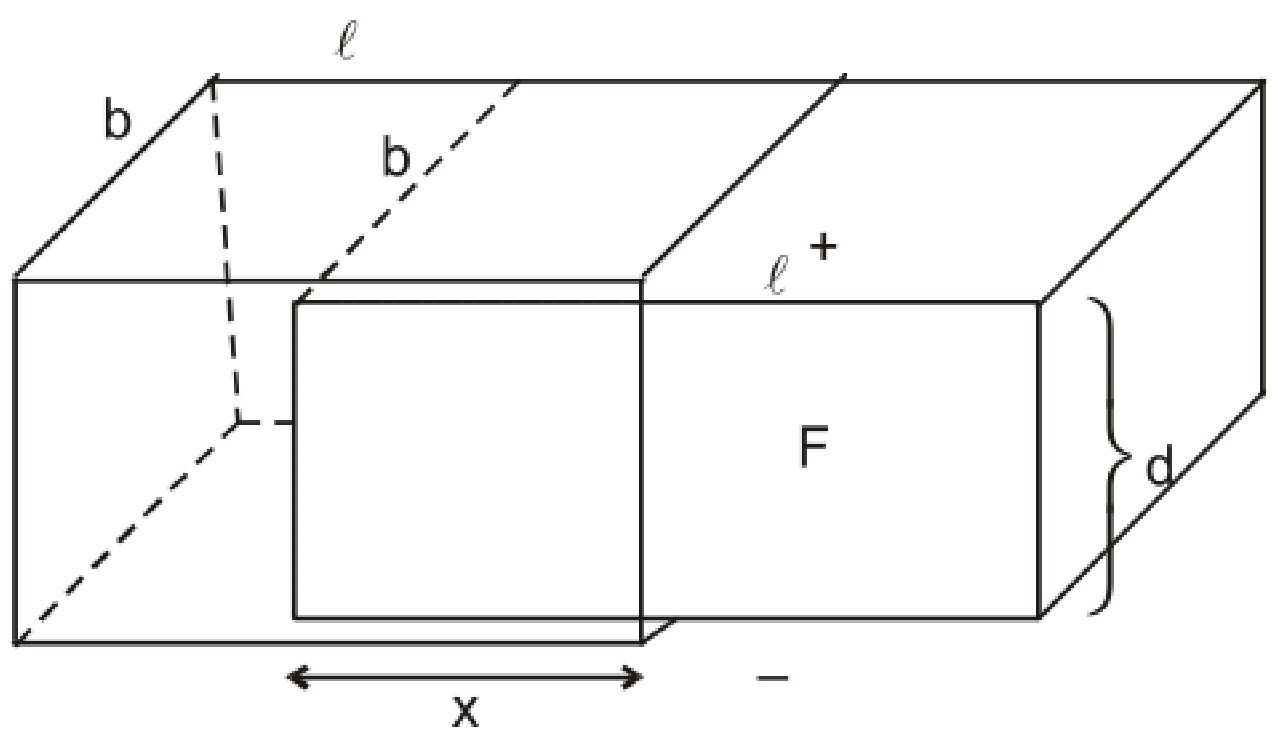
(ii) $$E_{in} = E - E_{ind} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_b}{\varepsilon_0} = \frac{\sigma}{K\varepsilon_0} = \frac{V}{d}$$

$E : \frac{\sigma}{\varepsilon_0}$ Electric field in the absence of dielectric

E_{ind} : Induced (bound) charge density.

(iii) $$\sigma_b = \sigma\left(1 - \frac{1}{K}\right).$$

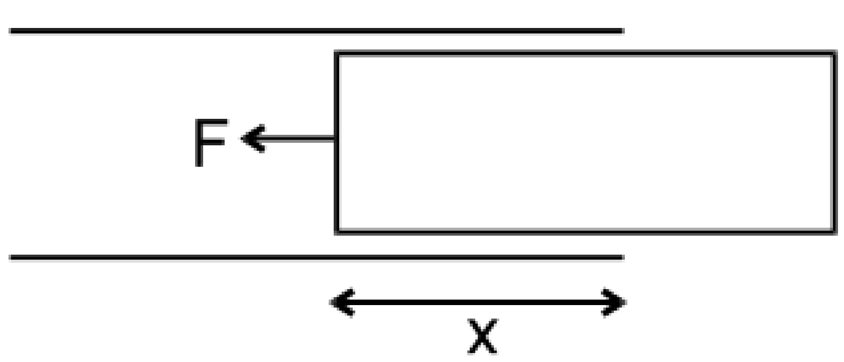
6. Force on dielectric



(i) When battery is connected

$$F = \frac{\varepsilon_0 b(K-1)V^2}{2d}$$

(ii) When battery is not connected $F = \dfrac{Q^2}{2C^2}\dfrac{dC}{dx}$



* Force on the dielectric will be zero when the dielectric is fully inside.