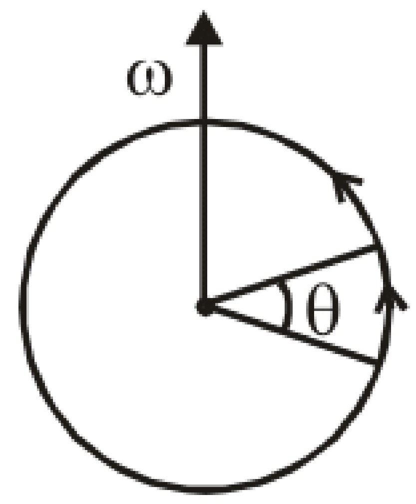
CIRCULAR MOTION

1. Average angular velocity $\Rightarrow$ $\omega_{av} = \dfrac{\theta_2 - \theta_1}{t_2 - t_1} = \dfrac{\Delta\theta}{\Delta t}$



2. Instantaneous angular velocity $\Rightarrow$ $\omega = \dfrac{d\theta}{dt}$

3. Average angular acceleration $\Rightarrow$ $\alpha_{av} = \dfrac{\omega_2 - \omega_1}{t_2 - t_1} = \dfrac{\Delta\omega}{\Delta t}$

4. Instantaneous angular acceleration $\Rightarrow$ $\alpha = \dfrac{d\omega}{dt} = \omega\dfrac{d\omega}{d\theta}$

5. Relation between speed and angular velocity $\Rightarrow$ $v = r\omega$ and $\vec{v} = \vec{\omega} \times \vec{r}$

7. Tangential acceleration (rate of change of speed)

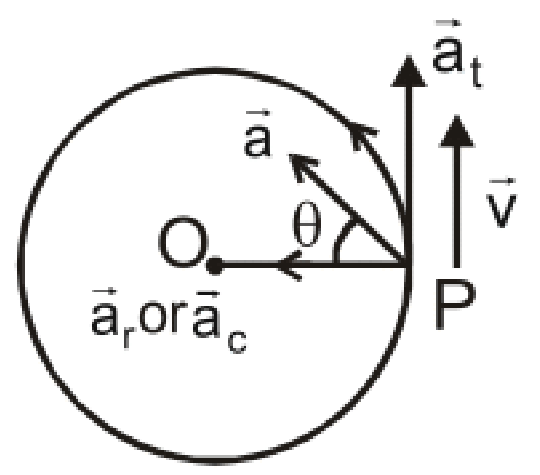
$$\Rightarrow \quad a_t = \frac{dV}{dt} = r\frac{d\omega}{dt} = \omega\frac{dr}{dt}$$

8. Radial or normal or centripetal acceleration $\Rightarrow$ $a_r = \dfrac{v^2}{r} = \omega^2 r$

9. Total acceleration

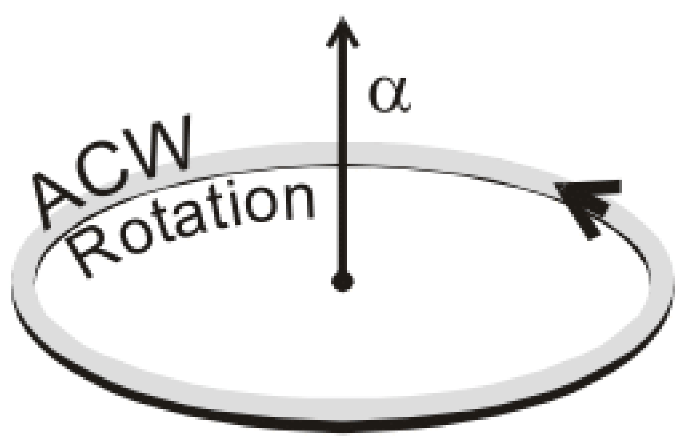
$$\Rightarrow \quad \vec{a} = \vec{a}_t + \vec{a}_r \Rightarrow a = (a_t^2 + a_r^2)^{1/2}$$

Where $\vec{a}_t = \vec{\alpha} \times \vec{r}$ and $\vec{a}_r = \vec{\omega} \times \vec{v}$



10. Angular acceleration

$$\Rightarrow \quad \vec{\alpha} = \frac{d\vec{\omega}}{dt} \text{ (Non-uniform circular motion)}$$

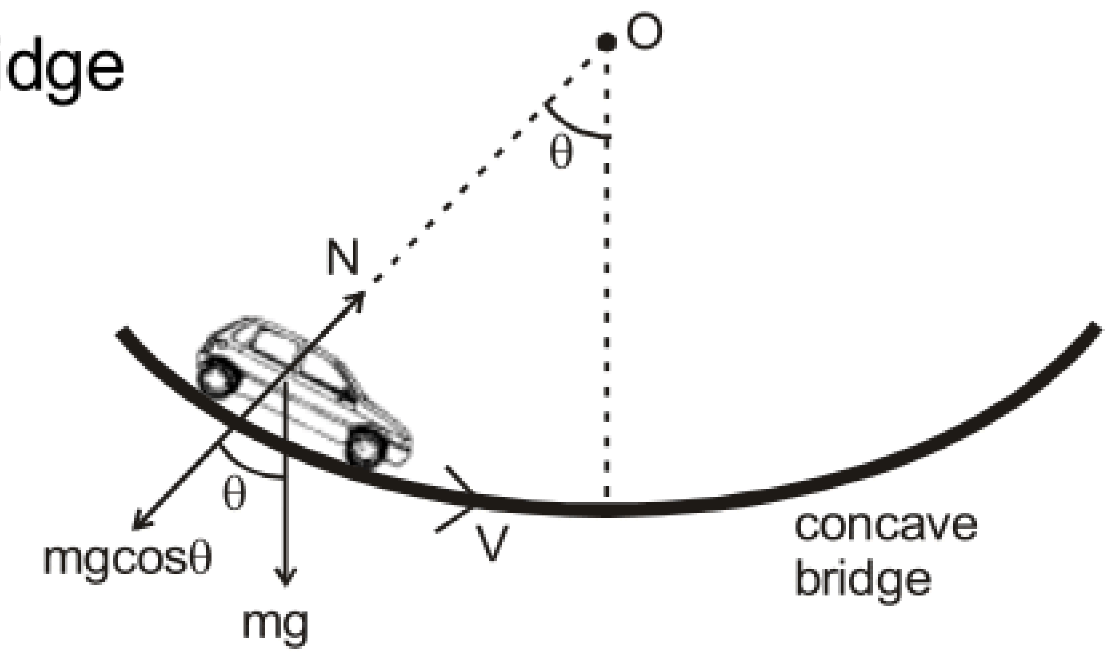


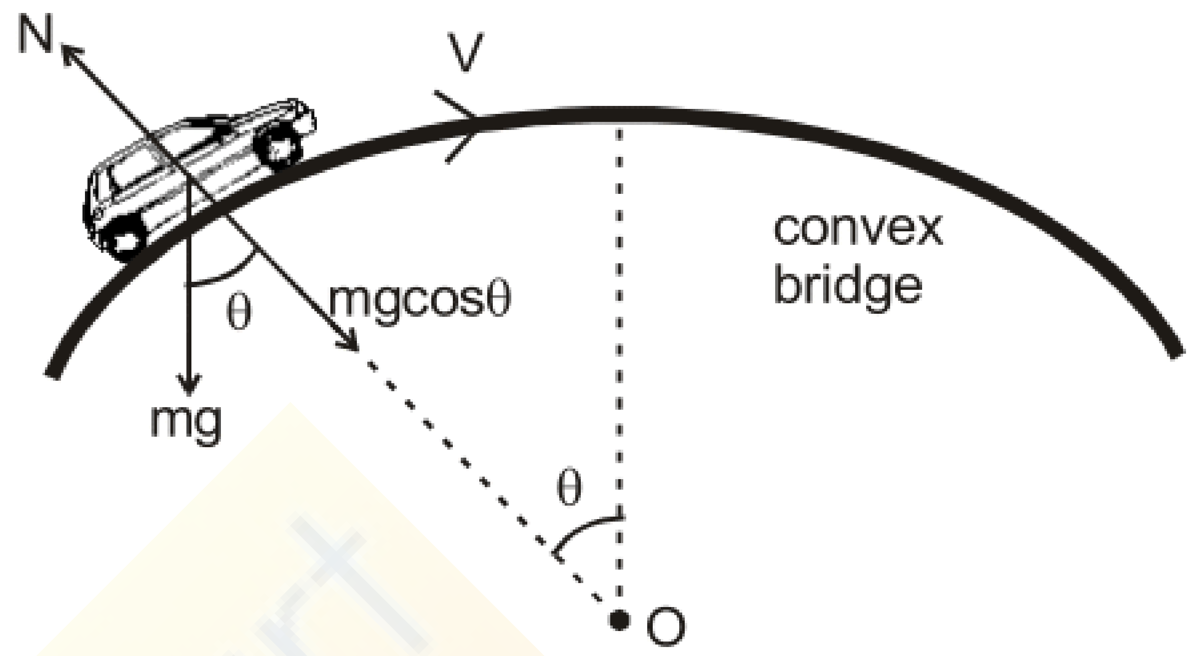
12. Radius of curvature $R = \dfrac{v^2}{a_\perp} = \dfrac{mv^2}{F_\perp}$

If y is a function of x. i.e. y = f(x) $\Rightarrow$ $R = \dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$

13. Normal reaction of road on a concave bridge

$$\Rightarrow \qquad N = mg\cos\theta + \frac{mv^2}{r}$$





14. Normal reaction on a convex bridge

$$\Rightarrow \qquad N = mg\cos\theta - \frac{mv^2}{r}$$

15. Skidding of vehicle on a level road $\Rightarrow$ $v_{safe} \le \sqrt{\mu g r}$

16. Skidding of an object on a rotating platform $\Rightarrow$ $\omega_{max} = \sqrt{\mu g / r}$

17. Bending of cyclist $\Rightarrow \tan\theta = \dfrac{v^2}{rg}$

18. Banking of road without friction $\Rightarrow \tan\theta = \dfrac{v^2}{rg}$

19. Banking of road with friction $\Rightarrow \dfrac{v^2}{rg} = \dfrac{\mu + \tan\theta}{1 - \mu\tan\theta}$

20. Maximum also minimum safe speed on a banked frictional road

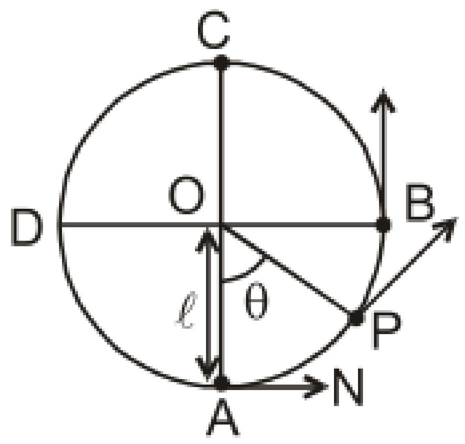
$$V_{max} = \left[\frac{rg(\mu + \tan\theta)}{(1 - \mu\tan\theta)}\right]^{1/2} \qquad V_{min} = \left[\frac{rg\ \ (\tan\theta - \mu)}{(1 + \mu\tan\theta)}\right]^{1/2}$$

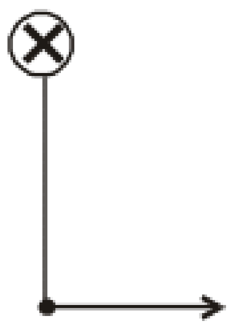
21. Centrifugal force (pseudo force) $\Rightarrow f = m\omega^2 r$, acts outwards when the particle itself is taken as a frame.

22. Effect of earths rotation on apparent weight $\Rightarrow N = mg - mR\omega^2\cos^2\theta$;

where $\theta \Rightarrow$ latitude at a place

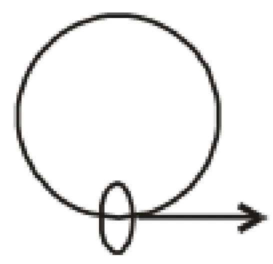
23. Various quantities for a critical condition in a vertical loop at different positions



(1) 

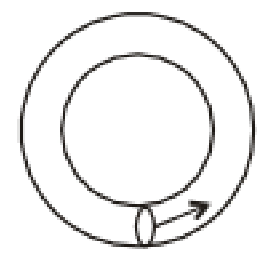
$V_{min} = \sqrt{4gL}$

(for completing the circle)

(2) 

$V_{min} = \sqrt{4gL}$

(for completing the circle)

(3) 

$V_{min} = \sqrt{4gL}$

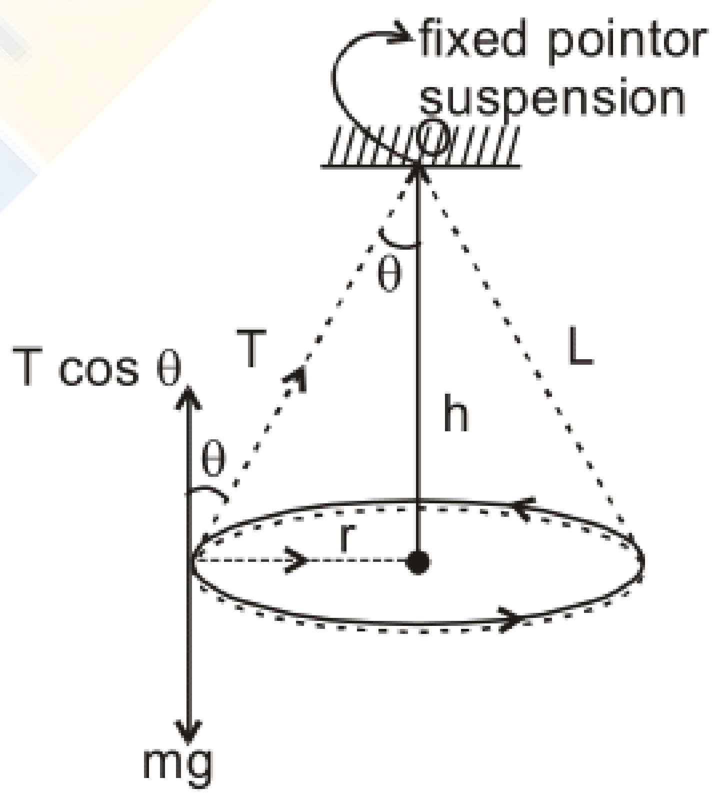
(for completing the circle)

24. Conical pendulum :

$T \cos \theta = mg$

$T \sin \theta = m\omega^2 r$

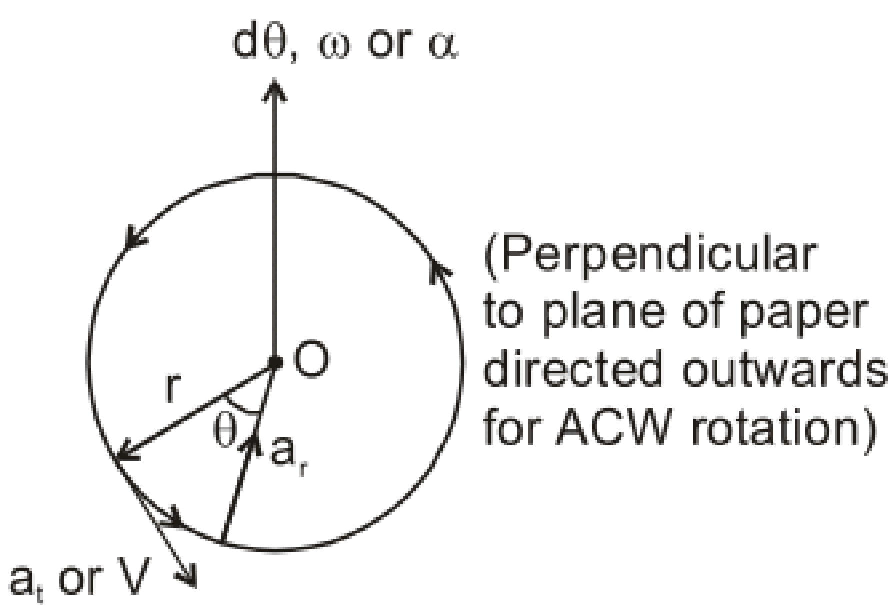
$\therefore$ Time period = $2\pi\sqrt{\dfrac{L\cos\theta}{g}}$



25. Relations amoung angular variables :

$\omega_0 \Rightarrow$ Initial ang. velocity

$\omega = \omega_0 + \alpha t$



$\omega \Rightarrow$ Find angular velocity

$\omega \Rightarrow$ Const. angular acceleration

$\theta \Rightarrow$ Angular displacement

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$