

## GRAVITATION : Universal Law of Gravitation

$$F \propto \frac{m_1 m_2}{r^2} \text{ or } F = G \frac{m_1 m_2}{r^2}$$

where  $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$  is the universal gravitational constant.

### Newton's Law of Gravitation in vector form :

$$\vec{F}_{12} = \frac{Gm_1 m_2}{r^2} \hat{r}_{12} \quad \& \quad \vec{F}_{21} = \frac{Gm_1 m_2}{r^2} \hat{r}_{21}$$

Now  $\hat{r}_{12} = -\hat{r}_{21}$ , Thus  $\vec{F}_{21} = \frac{-Gm_1 m_2}{r^2} \hat{r}_{12}$ .

Comparing above, we get  $\vec{F}_{12} = -\vec{F}_{21}$

**Gravitational Field**  $E = \frac{F}{m} = \frac{GM}{r^2}$

**Gravitational potential** : gravitational potential,

$$V = -\frac{GM}{r} . \quad E = -\frac{dV}{dr} .$$

**1. Ring.**  $V = \frac{-GM}{x \text{ or } (a^2 + r^2)^{1/2}}$  &  $E = \frac{-GMr}{(a^2 + r^2)^{3/2}} \hat{r}$  or  $E = -\frac{GM \cos \theta}{x^2}$

Gravitational field is maximum at a distance,

$$r = \pm a/\sqrt{2} \text{ and it is } -2GM/3\sqrt{3}a^2$$

### 2. Thin Circular Disc.

$$V = \frac{-2GM}{a^2} \left[ \left[ a^2 + r^2 \right]^{1/2} - r \right] \quad \& \quad E = -\frac{2GM}{a^2} \left[ 1 - \frac{r}{\left[ r^2 + a^2 \right]^{1/2}} \right] = -\frac{2GM}{a^2} [1 - \cos \theta]$$

### 3. Non conducting solid sphere

(a) Point P inside the sphere.  $r \leq a$ , then

$$V = -\frac{GM}{2a^3} (3a^2 - r^2) \quad \& \quad E = -\frac{GMr}{a^3}, \text{ and at the centre } V = -\frac{3GM}{2a} \text{ and } E = 0$$

(b) Point P outside the sphere .

$$r \geq a, \text{ then } V = -\frac{GM}{r} \quad \& \quad E = -\frac{GM}{r^2}$$

### 4. Uniform Thin Spherical Shell / Conducting solid sphere

(a) Point P Inside the shell.

$$r \leq a, \text{ then } V = \frac{-GM}{a} \quad \& \quad E = 0$$

(b) Point P outside shell.

$$r \geq a, \text{ then } V = \frac{-GM}{r} \quad \& \quad E = -\frac{GM}{r^2}$$

## VARIATION OF ACCELERATION DUE TO GRAVITY :

### 1. Effect of Altitude

$$g_h = \frac{GM_e}{(R_e + h)^2} = g \left(1 + \frac{h}{R_e}\right)^{-2} \approx g \left(1 - \frac{2h}{R_e}\right) \text{ when } h \ll R.$$

$$2. \text{ Effect of depth} \quad g_d = g \left(1 - \frac{d}{R_e}\right)$$

### 3. Effect of the surface of Earth

The equatorial radius is about 21 km longer than its polar radius.

We know,  $g = \frac{GM_e}{R_e^2}$  Hence  $g_{\text{pole}} > g_{\text{equator}}$ .

## SATELLITE VELOCITY (OR ORBITAL VELOCITY)

$$v_0 = \left[ \frac{GM_e}{(R_e + h)} \right]^{\frac{1}{2}} = \left[ \frac{gR_e^2}{(R_e + h)} \right]^{\frac{1}{2}}$$

When  $h \ll R_e$  then  $v_0 = \sqrt{gR_e}$

$$\therefore v_0 = \sqrt{9.8 \times 6.4 \times 10^6} = 7.92 \times 10^3 \text{ ms}^{-1} = 7.92 \text{ km s}^{-1}$$

### Time period of Satellite

$$T = \frac{2\pi(R_e + h)}{\left[ \frac{gR_e^2}{(R_e + h)} \right]^{\frac{1}{2}}} = \frac{2\pi}{R_e} \left[ \frac{(R_e + h)^3}{g} \right]^{\frac{1}{2}}$$

### Energy of a Satellite

$$U = \frac{-GM_e m}{r} \quad \text{K.E.} = \frac{GM_e m}{2r}; \text{ then total energy} \rightarrow E = -\frac{GM_e m}{2R_e}$$

### Kepler's Laws

#### Law of area :

The line joining the sun and a planet sweeps out equal areas in equal intervals of time.

$$\text{Areal velocity} = \frac{\text{area swept}}{\text{time}} = \frac{\frac{1}{2}r(rd\theta)}{dt} = \frac{1}{2}r^2 \frac{d\theta}{dt} = \text{constant}.$$

$$\text{Hence } \frac{1}{2}r^2\omega = \text{constant. Law of periods : } \frac{T^2}{R^3} = \text{constant}$$