SIMPLE HARMONIC MOTION

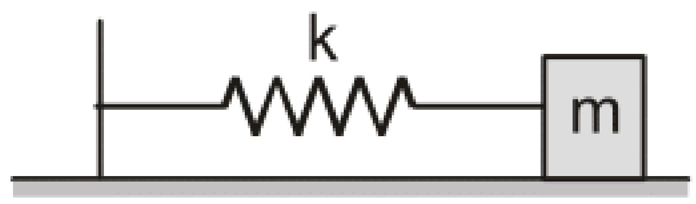
S.H.M.

$F = -kx$

General equation of S.H.M. is $x = A\sin(\omega t + \phi)$; $(\omega t + \phi)$ is phase of the motion and ϕ is initial phase of the motion.

Angular Frequency (ω) : $\omega = \dfrac{2\pi}{T} = 2\pi f$

Time period (T) : $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$



Speed : $v = \omega\sqrt{A^2 - x^2}$

Acceleration : $a = -\omega^2 x$

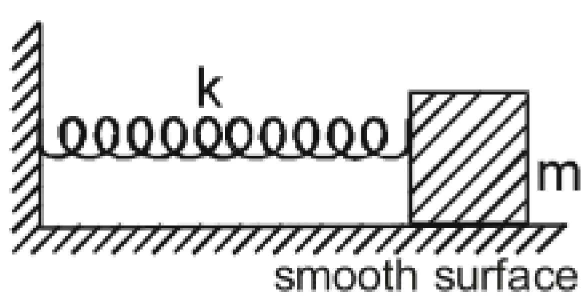
Kinetic Energy (KE) : $\dfrac{1}{2}mv^2 = \dfrac{1}{2}m\omega^2(A^2 - x^2) = \dfrac{1}{2}k(A^2 - x^2)$

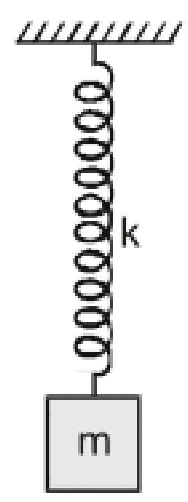
Potential Energy (PE) : $\dfrac{1}{2}Kx^2$

Total Mechanical Energy (TME)

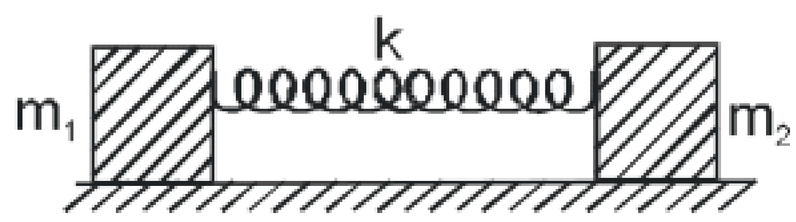
$= \text{K.E.} + \text{P.E.} = \dfrac{1}{2}k(A^2 - x^2) + \dfrac{1}{2}Kx^2 = \dfrac{1}{2}KA^2$ (which is constant)

SPRING-MASS SYSTEM

(1) 

⇒ 

⇒ $T = 2\pi\sqrt{\dfrac{m}{k}}$

(2) 

$T = 2\pi\sqrt{\dfrac{\mu}{K}}$, where $\mu = \dfrac{m_1 m_2}{(m_1 + m_2)}$ known as reduced mass

COMBINATION OF SPRINGS

Series Combination : $1/k_{eq} = 1/k_1 + 1/k_2$

Parallel combination : $k_{eq} = k_1 + k_2$

SIMPLE PENDULUM $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{g_{eff.}}}$ (in accelerating Reference Frame); g_{eff} is net acceleration due to pseudo force and gravitational force.

COMPOUND PENDULUM / PHYSICAL PENDULUM

Time period (T) : $T = 2\pi\sqrt{\frac{I}{mg\ell}}$

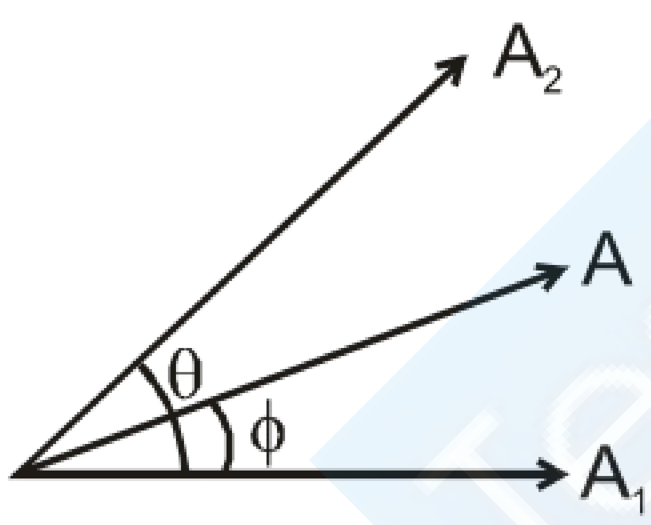
where, $I = I_{CM} + m\ell^2$; ℓ is distance between point of suspension and centre of mass.

TORSIONAL PENDULUM

Time period (T) : $T = 2\pi\sqrt{\frac{I}{C}}$ where, C = Torsional constant

Superposition of SHM's along the same direction

$x_1 = A_1 \sin \omega t$ & $x_2 = A_2 \sin (\omega t + \theta)$



If equation of resultant SHM is taken as $x = A \sin (\omega t + \phi)$

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$ & $\tan\phi = \frac{A_2 \sin\theta}{A_1 + A_2\cos\theta}$

1. **Damped Oscillation**

- **Damping force**

$\vec{F} = -b\vec{v}$

- **equation of motion is**

$\frac{mdv}{dt} = -kx - bv$

- $b^2 - 4mK > 0$ over damping

- $b^2 - 4mK = 0$ critical damping
- $b^2 - 4mK < 0$ under damping
- For small damping the solution is of the form.

$$x = \left(A_0 e^{-bt/2m}\right) \sin [\omega' t + \delta], \text{ where } \omega' = \sqrt{\left(\frac{k}{m}\right) - \left(\frac{b}{2m}\right)^2}$$

For small b

- angular frequency $\omega' \approx \sqrt{k/m,} = \omega_0$

- Amplitude $A = A_0 e^{\frac{-bt}{2m}}$

- Energy $E(t) = \frac{1}{2} KA^2 \, e^{-bt/m}$

- Quality factor or Q value , $Q = 2\pi \frac{E}{|\Delta E|} = \frac{\omega'}{2\omega_Y}$

where , $\omega' = \sqrt{\frac{k}{m} . \frac{b^2}{4m^2}}$, $\omega_Y = \frac{b}{2m}$

2. Forced Oscillations And Resonance

External Force $F(t) = F_0 \cos \omega_d t$

$x(t) = A \cos (\omega_d t + \phi)$

$$A = \frac{F_0}{\sqrt{\left(m^2 \left(\omega^2 - \omega_d^2\right)^2 + \omega_d^2 b^2\right)}} \text{ and } \tan\phi = \frac{-v_0}{\omega_d x_0}$$

(a) Small Damping $A = \frac{F_0}{m\left(\omega^2 - \omega_d^2\right)}$

(b) Driving Frequency Close to Natural Frequency $A = \frac{F_0}{\omega_d b}$