STRING WAVES

GENERAL EQUATION OF WAVE MOTION :

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

$y(x,t) = f\left(t \pm \frac{x}{v}\right)$

where, y (x, t) should be finite everywhere.

$\Rightarrow$ $f\left(t + \frac{x}{v}\right)$ represents wave travelling in – ve x-axis.

$\Rightarrow$ $f\left(t - \frac{x}{v}\right)$ represents wave travelling in + ve x-axis.

$y = A \sin(\omega t \pm kx + \phi)$

TERMS RELATED TO WAVE MOTION (FOR 1-D PROGRESSIVE SINE WAVE)

(e) Wave number (or propagation constant) (k) :

$k = 2\pi/\lambda = \frac{\omega}{v}$ (rad m^{-1})

(f) Phase of wave : The argument of harmonic function $(\omega t \pm kx + \phi)$ is called phase of the wave.

Phase difference $(\Delta\phi)$: difference in phases of two particles at any time t.

$\Delta\phi = \frac{2\pi}{\lambda}\,\Delta x$ Also. $\Delta\phi = \frac{2\pi}{T}.\,\Delta t$

SPEED OF TRANSVERSE WAVE ALONG A STRING/WIRE.

$v = \sqrt{\frac{T}{\mu}}$ where T = Tension, μ = mass per unit length

POWER TRANSMITTED ALONG THE STRING BY A SINE WAVE

Average Power $\langle P \rangle = 2\pi^2 f^2 A^2 \mu v$

Intensity $I = \frac{\langle P \rangle}{s} = 2\pi^2 f^2 A^2 \rho v$

REFLECTION AND REFRACTION OF WAVES

$y_i = A_i \sin(\omega t - k_1 x)$

$y_t = A_t \sin(\omega t - k_2 x)$
$y_r = -A_r \sin(\omega t + k_1 x)$
if incident from rarer to denser medium $(v_2 < v_1)$

$$\left.\begin{array}{l} y_t = A_t \sin(\omega t - k_2 x) \\ y_r = A_r \sin(\omega t + k_1 x) \end{array}\right] \text{ if incident from denser to rarer medium. } (v_2 > v_1)$$

(d) Amplitude of reflected & transmitted waves.

$$A_r = \frac{|k_1 - k_2|}{k_1 + k_2} A_i \ \ \& \ \ A_t = \frac{2k_1}{k_1 + k_2} A_i$$

STANDING/STATIONARY WAVES :-

(b) $y_1 = A \sin(\omega t - kx + \theta_1)$
$y_2 = A \sin(\omega t + kx + \theta_2)$

$$y_1 + y_2 = \left[2A\cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)\right] \sin\left(\omega t + \frac{\theta_1 + \theta_2}{2}\right)$$

The quantity $2A\cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)$ represents resultant amplitude at x. At some position resultant amplitude is zero these are called **nodes**. At some positions resultant amplitude is 2A, these are called **antinodes.**

(c) Distance between successive nodes or antinodes = $\frac{\lambda}{2}$.

(d) Distance between successive nodes and antinodes = $\lambda/4$.

(e) All the particles in same segment (portion between two successive nodes) vibrate in same phase.

(f) The particles in two consecutive segments vibrate in opposite phase.

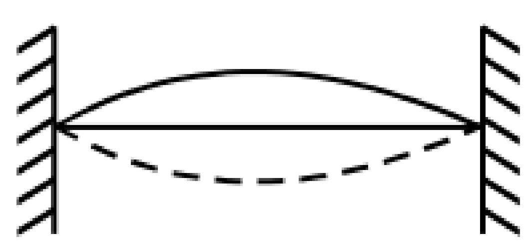
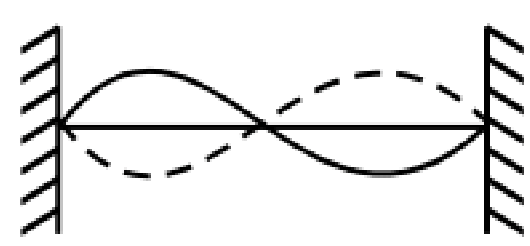
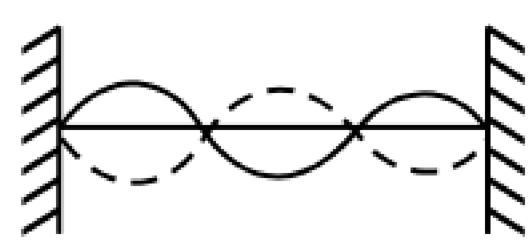
(g) Since nodes are permanently at rest so energy can not be transmitted across these.

VIBRATIONS OF STRINGS (STANDING WAVE)

(a) Fixed at both ends :

1. Fixed ends will be nodes. So waves for which

$L = \frac{\lambda}{2}$ $\qquad$ $L = \frac{2\lambda}{2}$ $\qquad$ $L = \frac{3\lambda}{2}$

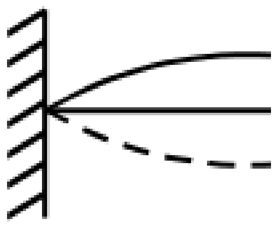
are possible giving

$$L = \frac{n\lambda}{2} \qquad \text{or } \lambda = \frac{2L}{n} \quad \text{where } n = 1, 2, 3, ...$$

$$\text{as} \quad v = \sqrt{\frac{T}{\mu}} \qquad f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}\ , \ n = \text{no. of loops}$$

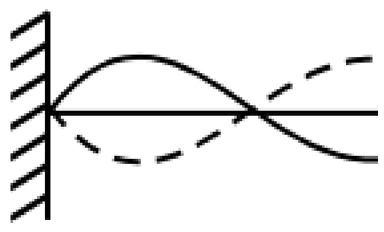
(b) String free at one end :

1. for fundamental mode L = $\frac{\lambda}{4}$ = or $\lambda = 4L$



fundamental mode

First overtone L = $\frac{3\lambda}{4}$ Hence $\lambda = \frac{4L}{3}$

$\Rightarrow$ 

first overtone

so $f_1 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$ (First overtone)

Second overtone $f_2 = \frac{5}{4L}\sqrt{\frac{T}{\mu}}$

so $f_n = \frac{\left(n+\frac{1}{2}\right)}{2L}\sqrt{\frac{T}{\mu}} = \frac{(2n+1)}{4L}\sqrt{\frac{T}{\mu}}$