Bar magnet

The iron ore magnetite which attracts small pieces of iron, cobalt, nickel etc. is a natural magnet. The natural magnets have irregular shape and they are weak. A piece of iron or steel acquires magnetic properties when it is rubbed with a magnet. Such magnets made out of iron or steel are artificial magnets. Artificial magnets can have desired Shape and desired strength. If the artificial magnet is in the form of a rectangular or cylindrical bar, it is called a bar magnet.

Basic properties of magnets

(i) When the magnet is dipped in iron filings, they cling to the ends of the magnet. The attraction is maximum at the two ends of the magnet. These ends are called poles of the magnet.
(ii) When a magnet is freely suspended, it always points along north-south direction. The pole pointing towards geographic north is called North Pole *N* and the pole which points towards geographic south is called South Pole *S.*
(iii) Magnetic poles always exist in pairs. (i.e) isolated magnetic pole does not exist.
(iv) The magnetic length of a magnet is always less than its geometric length, because the poles are situated a little inwards from the free ends of the magnet. (But for the purpose of calculation the geometric length is always taken as magnetic length.)
(v) Like poles repel each other and unlike poles attract each other.
North Pole of a magnet when brought near North Pole of another magnet, We can observe repulsion, but when the north pole of one magnet is brought near South Pole of another magnet, we observe attraction.
(vi) The force of attraction or repulsion between two magnetic poles is given by Coulomb's inverse square law.

Magnetic field

Magnetic field is the space in which a magnetic pole experiences a force or it is the space around a magnet in which the influence of the magnet is felt.

Magnetic lines of force

A magnetic field is better studied by drawing as many numbers of magnetic lines of force as possible. A magnetic line of force is a line along which a free isolated north pole would travel when it is placed in the magnetic field.

Properties of magnetic lines of force

(i) Magnetic lines of forces are closed continuous curves, extending through the body of the magnet.
(ii) The direction of line of force is from North Pole to South Pole outside the magnet. While it is from South Pole to North Pole inside the magnet.
(iii) The tangent to the magnetic line of force at any point gives the direction of magnetic field at that point. (i.e) it gives the direction of magnetic induction ($\vec{B}$) at that point.
(iv) They never intersect each other.
(v) They crowd where the magnetic field is strong and thin out where the field is weak.

Magnetic moment

Since any magnet has two poles, it is also called a magnetic dipole. The magnetic moment of a magnet is defined **as the product of the pole strength and the distance between the two poles.** If m is the pole strength of each pole and 2l is the distance
between the poles, the magnetic moment Magnetic moment is a vector quantity. It is denoted by M. Its unit is A m^2. Its direction is from south pole to north pole

$$\vec{M} = m(2\vec{l})$$

Bar magnet as an equivalent solenoid

The magnetic dipole moment m associated with a current loop was defined to be m=NIA where N is the number of turns in the loop, I the current and A the area vector. The direction of magnetic moment m of a loop can be found by using right hand rule, curl fingers in the direction of current then thumb gives the direction of magnetic moment. The resemblance of magnetic field lines for a bar magnet and a solenoid suggest that a bar magnet may be thought of as a large number of circulating currents in analogy with a solenoid.
Let the solenoid consists of n turns per unit length. Let its length be 2l and radius a. We can evaluate the axial field at a point P, at a distance r from the centre O of the solenoid. To do this, consider a circular element of thickness dx of the solenoid at a distance x from its centre. It consists of n d x turns. Let I be the current in the solenoid. The magnetic field on the axis of a circular current loop at point P due to the circular element is

$$dB = \frac{\mu_0 n dx I a^2}{2[(r-x)^2 + a^2]^{3/2}}$$

The magnitude of the total field is obtained by summing over all the elements — in other words by integrating from x = – l to x = + l . Thus,

$$B = \frac{\mu_0 n I a^2}{2} \int_{-l}^{+l} \frac{dx}{2[(r-x)^2 + a^2]^{3/2}}$$

Consider the far axial field of the solenoid, i.e., r >> a and r >> l . Then the denominator is approximated by

$$2[(r-x)^2 + a^2]^{3/2} = r^3$$

Then

$$B = \frac{\mu_0 n I a^2}{2r^3} \int_{-l}^{+l} dx$$

$$B = \frac{\mu_0 n I a^2}{2r^3}(2l)$$

Note that the magnitude of the magnetic moment of the solenoid is,
m = n (2l) I (πa^2) = (total number of turns × current × cross-sectional area).
Thus,

$$B = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$$

This is also the far axial magnetic field of a bar magnet which one may obtain experimentally. Thus, a bar magnet and a solenoid produce similar magnetic fields. The magnetic moment of a bar magnet is thus equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

Magnetic induction
Magnetic induction is the fundamental character of a magnetic field at a point. Magnetic induction at a point in a magnetic field is the force experienced by unit north pole placed at that point. It is denoted by B. Its unit is N/Am.
It is a vector quantity. It is also called as magnetic flux density. If a magnetic pole of strength m placed at a point in a magnetic field experiences a force F, the magnetic induction at that point is

$$\vec{B} = \frac{\vec{F}}{m}$$

Magnetic flux and magnetic flux density
The number of magnetic lines of force passing through an area A is called magnetic flux. It is denoted by φ. Its unit is Weber. It is a scalar quantity.

The number of magnetic lines of force crossing unit area kept normal to the direction of line of force is magnetic flux density. Its unit is Wb m^{-2} or Tesla.
Magnetic flux $\phi = \vec{B} \cdot \vec{A}$

Uniform and non-uniform magnetic field
Magnetic field is said to be uniform if the magnetic induction has the same magnitude and the same direction at all the points in the region. It is represented by drawing parallel lines
If the magnetic induction varies in magnitude and direction at different points in a region, the magnetic field is said to be non-uniform. The magnetic field due to a bar magnet is non-uniform. It is represented by convergent or divergent lines

Force between two magnetic poles
In 1785, Coulomb made use of his torsion balance and discovered the law governing the force between the two magnetic poles.

Coulomb's inverse square law
Coulomb's inverse square law states that the force of attraction or repulsion between the two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.
If m_1 and m_2 are the pole strengths of two magnetic poles separated by a distance of d in a medium, then
$F \propto m_1 m_2$ and $F \propto \frac{1}{d^2}$

$$F \propto \frac{m_1 m_2}{d^2}$$

$$F = K\frac{m_1 m_2}{d^2}$$

where K is the constant of proportionality and

$$K = \frac{\mu}{4\pi}$$

where μ is the permeability of the medium. But $\mu = \mu_0 \times \mu_r$
μ_0 - permeability of free space or vacuum.
μ_r - relative permeability of the medium
Let $m_1 = m_2 = 1$, and d = 1 m

$$K = \frac{\mu}{4\pi}$$

In free space, $\mu_o = 4\pi \times 10^{-7}$ H m^{-1}

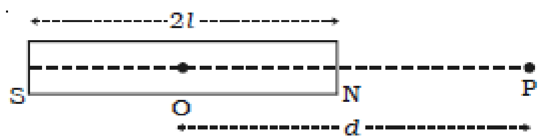
$$F = 10^{-7}\ \frac{m_1 m_2}{d^2}$$

$$F = 10^{-7}\ \frac{1 \times 1}{1}$$

F = 10-7 N

Therefore, unit pole is defined as that pole which when placed at a distance of 1 metre in free space or air from an equal and similar pole, repels it with a force of 10^{-7} N.

Magnetic induction at a point along the axial line due to a magnetic dipole (Bar magnet)
NS is the bar magnet of length 2l and of pole strength m. P is a point on the axial line at a distance d from its midpoint O



According to inverse square law,

$$F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{d^2}$$

Magnetic induction (B_1) at P due to north pole of the magnet,
Along NP

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(NP)^2}$$

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(d-l)^2}$$

Magnetic induction (B_2) at P due to south pole of the magnet,
Along PS

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{(PS)^2}$$

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{(d+l)^2}$$

Magnetic induction at *P* due to the bar magnet,

$$B = B_1 - B_2$$

$$B = \frac{\mu_0}{4\pi}\frac{m}{(d-l)^2} - \frac{\mu_0}{4\pi}\frac{m}{(d+l)^2}$$

$$B = \frac{\mu_0 m}{4\pi}\left(\frac{4ld}{(d^2-l^2)^2}\right)$$

$$B = \frac{\mu_0 m}{4\pi}\left(\frac{2l \times 2d}{(d^2-l^2)^2}\right)$$

$$B = \frac{\mu_0}{4\pi}\left(\frac{2l \times M}{(d^2-l^2)^2}\right)$$

As $M = m \times 2l$

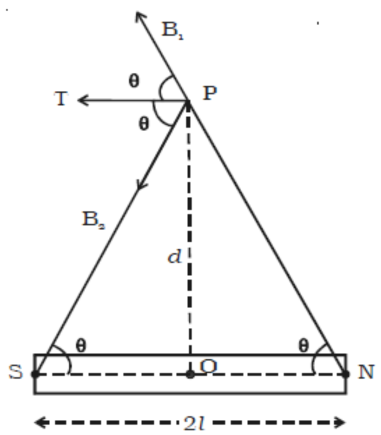
For a short bar magnet, l is very small compared to *d*, hence l^2 is neglected

$$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

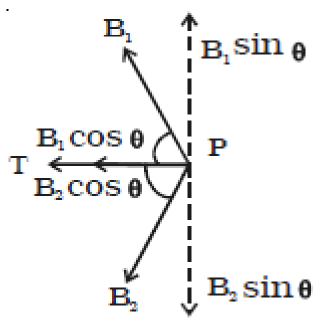
The direction of B is along the axial line away from the north pole.

Magnetic induction at a point along the equatorial line of a bar magnet

NS is the bar magnet of length 2*l* and pole strength m. P is a point on the equatorial line at a distance d from its midpoint O



Components magnetic field at point P are as follows



Magnetic induction (B_1) at P due to north pole of the magnet,
Along NP

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(NP)^2}$$

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(d^2 + l^2)}$$

(AS $NP^2 = NO^2 + OP^2$)
Magnetic induction ($B2$) at P due to south pole of the magnet,
Along PS

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{(PS)^2}$$

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{(d^2 + l^2)}$$

Resolving B_1 and B_2 into their horizontal and vertical components. Vertical components B_1 sin θ and B_2 *sin* θ are equal and opposite and therefore cancel each other
The horizontal components B_1 cos θ and B_2 cos θ will get added along PT.
Resultant magnetic induction at P due to the bar magnet is
$B = B_1 \cos\theta + B_2 \cos\theta$. (along PT)

$$B = \frac{\mu_0}{4\pi}\frac{m}{(d^2 + l^2)}\frac{l}{\sqrt{d^2 + l^2}} + \frac{\mu_0}{4\pi}\frac{m}{(d^2 + l^2)}\frac{l}{\sqrt{d^2 + l^2}}$$

$$cos\theta = \frac{SO}{PS} = \frac{NO}{NP}$$

As M =2lm

$$B = \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}}$$

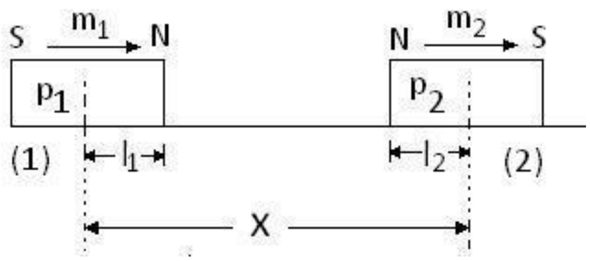
For a short bar magnet, l^2 is neglected.

$$B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

The direction of 'B' is along PT parallel to NS.

Solved numerical

Q) Find the force between two small bar magnets of magnetic moments m_1 and m_2 lying on the axis, as shown in figure (p_1 and p_2 are the pole strength of magnet (1) and (2) respectively, X is far greater than l_1 and l_2)



Solution :

To calculate force on magnet (2) due to magnet (1)

We will calculate magnetic field due to magnet (1) at the poles of the magnet(2).

Magnet (2) is on the axis of magnet (1).

Magnetic field at North pole of magnet (2), magnetic moment of magnet(1) is m_1

$$B_N = \frac{\mu_0}{4\pi}\frac{2m_1}{(x-l_2)^3}$$

The repulsive force F_N acting on the north pole of magnet(2) having pole strength p_2

$$F_N = p_2 B_N = \frac{\mu_0}{4\pi}\frac{2m_1 p_2}{(x-l_2)^3}$$

Similarly magnetic field at South pole of magnet(2), is

$$B_S = \frac{\mu_0}{4\pi}\frac{2m_1}{(x+l_2)^3}$$

The attractive force F_S acting on the north pole of magnet(2) having pole strength p_2

$$F_S = p_2 B_S = \frac{\mu_0}{4\pi}\frac{2m_1 p_2}{(x+l_2)^3}$$

Hence resultant force on magnet(2) is

$F=F_N - F_S$

$$F = \frac{\mu_0}{4\pi}2p_2 m_1\left[\frac{1}{(x-l_2)^3} - \frac{1}{(x+l_2)^3}\right]$$

$$F = \frac{\mu_0}{4\pi}2p_2 m_1\left[\frac{6x^2 l_2}{(x^2-l_2^2)^3}\right]$$

$$F = \frac{\mu_0}{4\pi}2p_2 m_1\left[\frac{6x^2 l_2}{(x^2)^3}\right]$$

$$F = \frac{\mu_0 m_1}{2\pi}\left[\frac{(2p_2 l_2)(3x^2)}{x^6}\right]$$

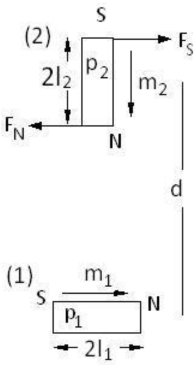
As $p_2 l_2 = m_2$

$$F = \frac{\mu_0 m_1}{2\pi}\left[\frac{(m_2)(3)}{x^4}\right]$$

$$F = \frac{3\mu_0 m_1 m_2}{2\pi x^4}$$

Resultant force is repulsive and acts on magnet (2) in a direction away from magnet(1)

Q) Find the torque on small bar magnet(2) due to small bar magnet(1), when they are placed perpendicular to each other as shown in figure l_2 and l_1 are far less than d



Solution :

From the figure Magnetic field at north pole of second magnet due to magnet(1) is

$$B_N = \frac{\mu_0}{4\pi}\frac{2m_1}{(d-l_2)^3}$$

Force on north pole is towards left is

$$F_N = p_2 B_N = \frac{\mu_0}{4\pi}\frac{2m_1 p_2}{(d-l_2)^3}$$

As X >> l_2

$$F_N = \frac{\mu_0}{4\pi}\frac{2m_1 p_2}{(d)^3}$$

Magnetic field at south pole of second magnet due to magnet(1) is

$$B_S = \frac{\mu_0}{4\pi}\frac{2m_1}{(d)^3}$$

Force on south pole is towards right is

$$F_S = p_2 B_S = \frac{\mu_0}{4\pi}\frac{2m_1 p_2}{(d+l_2)^3}$$

As X >> l_2

$$F_S = \frac{\mu_0}{4\pi}\frac{2m_1 p_2}{(d)^3}$$

Since $F_N = F_S$ are non collinear, equal and opposite in direction, they form a couple. Hence the torque is produced

$$\vec{\tau} = \vec{F_N} \times 2\vec{l_2} = \vec{F_S} \times 2\vec{l_2}$$

Since F_N and F_S are perpendicular to l_2 magnitude of torque with respect to centre of magnet (2)

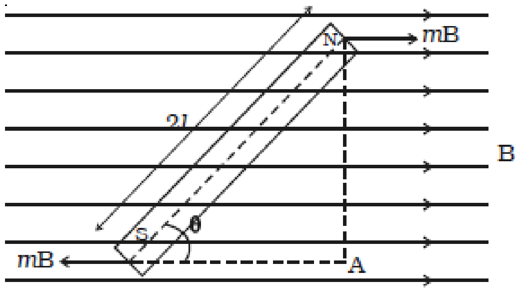
$$\tau = 2F_N l_2 = \frac{\mu_0}{4\pi}\frac{m_1 2l_2 p_2}{d^3}$$

as $2l_2p_2 = m_2$ magnetic dipole moment of magnet(2)

$$\tau = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{d^3}$$

Torque on a bar magnet placed in a uniform magnetic field

Consider a bar magnet *NS* of length 2*l* and pole strength m placed in a uniform magnetic field of induction *B* at an angle θ with the direction of the field



Due to the magnetic field B, a force mB acts on the north pole along the direction of the field and a force mB acts on the south pole along the direction opposite to the magnetic field.
These two forces are equal and opposite, hence constitute a couple. The torque τ due to the couple is
τ= one of the forces × perpendicular distance between them
τ = F × NA
τ = mB × NA ...(1)
τ = mB × 2*l* sin θ
τ = MB sin θ ...(2)
Vectorially,

$$\vec{\tau} = \vec{M} \times \vec{B}$$

The direction of τ is perpendicular to the plane containing $\vec{M}$ and $\vec{B}$
If $B = 1$ and $\theta = 90^o$
Then from equation (2), $\tau = M$
Hence, moment of the magnet M is equal to the torque necessary to keep the agent at right angles to a magnetic field of unit magnetic induction.

Periodic time of bar magnet
Here τ is restoring torque and θ is the angle between $\vec{M}$ and $\vec{B}$.
Now Newton's second law

$$\tau = I\frac{d^2\theta}{dt^2}$$

Here I is moment of inertia of bar magnet
Therefore, in equilibrium

$$I\frac{d^2\theta}{dt^2} = -mBsin\theta$$

Negative sign with mB sinθ implies that restoring torque is in opposition to deflecting torque. For small values of θ in radians, we approximate sin θ ≈ θ and get

$$I\frac{d^2\theta}{dt^2} = -mB\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta$$

This represents a simple harmonic motion. The square of the angular frequency is $\omega^2 = mB/I$ and the time period is

$$T = 2\pi\sqrt{\frac{I}{mB}}$$

The magnetic potential energy U_m
The magnetic potential energy U_m is given by

$$U_m = \int \tau d\theta$$

$$U_m = \int mBsin\theta d\theta$$

$$U_m = -mBcos\theta$$

$$U_m = \vec{m} \cdot \vec{B}$$

When the needle is perpendicular to the field, Equation shows that potential energy is minimum (= –mB) at θ = 0° (most stable position) and maximum (= +mB) at θ = 180° (most unstable position).

Solved numerical
Q) Work done in moving a magnet of magnetic moment m from most stable to most unstable position
<u>Solution:</u>
Most stable position is θ = 0° and most unstable position is θ = 180° hence work done
$W = U_B(\theta = 180^o) - U_B(\theta = 0^o) = mB - (-mB) = 2mB$

Q) A bar magnet is suspended horizontally by a torsion less wire in magnetic meridian. In order to deflect the magnet through 30° from the magnetic meridian, the upper end of the wire has to be rotated by 270°. Now this magnet is replaced by another magnet. In order to deflect the second magnet through the same angle from the magnetic meridian, the upper end of the wire has to be rotated by 180°. What is the ratio of the magnetic moments of the two bar magnets.

Solution

Let C be the deflecting torque per unit twist and M_1 and M_2 be the magnetic moments of the two magnets.

The deflecting torque is $\tau = C\theta$

The restoring torque is $\tau = MB \sin\theta$

In equilibrium,

deflecting torque = restoring torque

For the Magnet − I

$C(270^\circ - 30^\circ) = M_1 B_H \sin\theta$ ... (1)

For the magnet − II

$C(180^\circ - 30^\circ) = M_2 B_H \sin\theta$ ... (2)

Dividing (1) by (2)

$$\frac{M_1}{M_2} = \frac{240^o}{150^o} = \frac{8}{5}$$

Q) A magnetic needle placed in uniform magnetic field has magnetic moment 6.7×10^{-2} Am^2 and moment of inertia of 15×10^{-6} kgm^2. It performs 10 complete oscillations in 6.7 s. What is the magnitude of the magnetic field

Solution:

The periodic time of oscillation is

$$T = 2\pi\sqrt{\frac{I}{mB}}$$

$$B = 4\pi^2\frac{I}{mT^2}$$

$$B = \frac{4\pi^2(3.13)^2 \times 15 \times 10^{-6}}{6.7 \times 10^2 \times (0.67)^2} = 0.02T$$

Gauss's Law for Magnetic Field

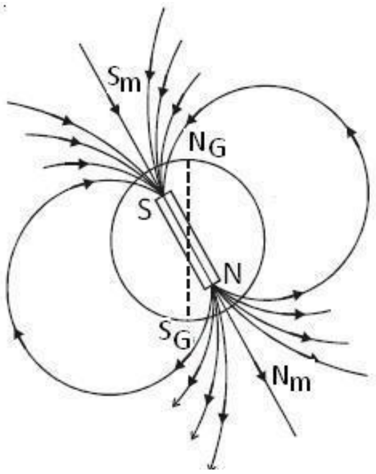
Magnetic field lines always forms a closed loops, the magnetic flux associated with any closed surface is always zero

$$\oint_{\substack{Closed\\Sufrace}} \vec{B}\cdot\overrightarrow{da} = 0$$

Where B is the magnetic field and ds is an infinitesimal area vector of the closed surface "The net magnetic flux passing through any closed surface is zero" This statement is called Gauss's law for magnetic field.

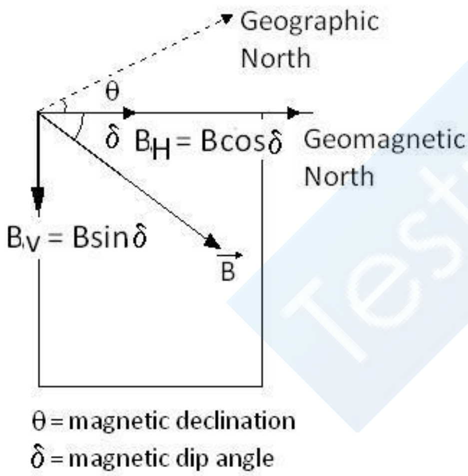
Earth's magnetic field and magnetic elements

A freely suspended magnetic needle at a point on Earth comes to rest approximately along the geographical north - south direction.



This shows that the Earth behaves like a huge magnetic dipole with its magnetic poles near its geographical poles. Since the north pole of the magnetic needle approximately points towards geographic north (NG) it is appropriate to call the magnetic pole near NG as the magnetic south pole of Earth Sm. Also, the pole near SG is the magnetic north pole of the Earth (Nm).
The Earth's magnetic field at any point on the Earth can be completely defined in terms of certain quantities called magnetic elements of the Earth, namely

(i) Declination or the magnetic variation θ.
The angle between the magnetic meridian and geographic meridian at a place on the surface of the earth is called magnetic declination at that place



(ii) Dip or inclination δ
Magnetic dip or angle of inclination is the angle δ (up or down) that the magnetic field of earth makes with the horizontal at a place in magnetic meridian
(iii) The horizontal and vertical component of the Earth's magnetic field.
B_V = B sinδ and B_H = Bcosδ

$$\tan\delta = \frac{B_V}{B_H}$$

$$B = \sqrt{B_V^2 + B_H^2}$$

Causes of the Earth's magnetism
The exact cause of the Earth's magnetism is not known even today. However, some important factors which may be the cause of Earth's magnetism are:

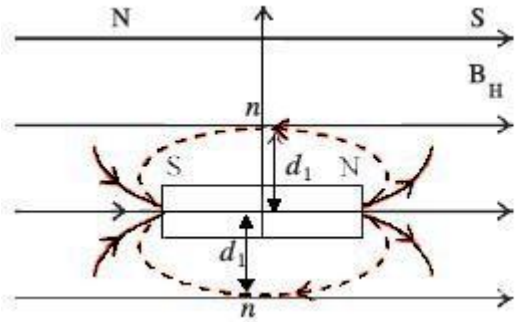
(i) Magnetic masses in the Earth.
(ii) Electric currents in the Earth.
(iii) Electric currents in the upper regions of the atmosphere.
(iv) Radiations from the Sun.
(v) Action of moon etc.
However, it is believed that the Earth's magnetic field is due to the molten charged metallic fluid inside the Earth's surface with a core of radius about 3500 km compared to the Earth's radius of 6400 km.

Solved Numerical

Q) A short bar magnet is placed with its north pole pointing north. The neutral point is 10 cm away from the centre of the magnet. If B = 4×10^{-5} T, calculate the magnetic moment of the magnet.

Solution:

When we keep North pole pointing north pole it means, it is in the direction of field lines of earth is opposite to magnetic field lines of magnet.
As shown in figure let neutral point (where effective magnetic field becomes zero) be at point n, at distance d_1 = 20 cm



Now magnetic field due to bar magnet = Horizontal component of earth

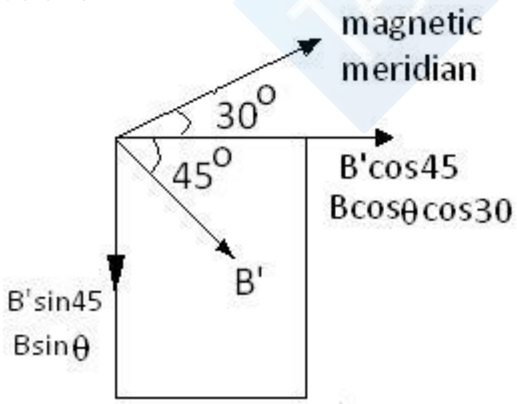
$$\frac{\mu_0}{4\pi}\frac{m}{d_1^3} = B_H$$

$$10^{-7}\frac{m}{(0.1)^3} = 4 \times 10^5$$

m=0.4 A m^2

Q) A magnet makes an angle of 45° with the horizontal in a plane making an angle of 30° with the magnetic meridian. Find the true value of the dip angle at the place.

Solution:



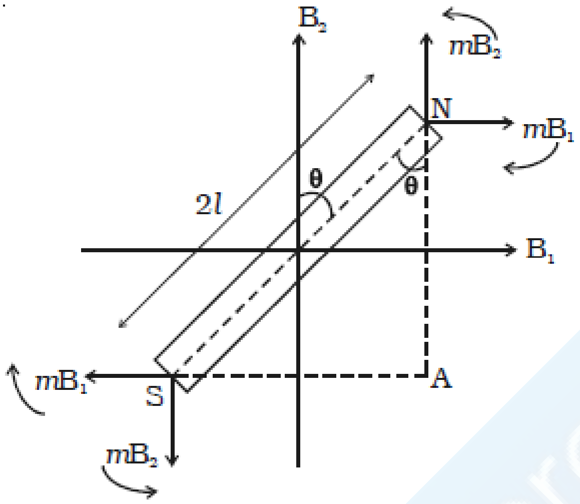
Let B be the magnetic field in magnetic meridian, making an angle of θ with horizontal.
Thus Horizontal component is B_H = Bcos θ and
vertical component is B_V = Bsinθ
Component of Horizontal component of magnetic field in magnetic meridian along plane = Bcosθcos30
Let magnetic field in plane be B'. Thus Horizontal component B'_H = B'cos45 and Vertical component B'_V = B'sin45
From above

$B\sin\theta = B'\sin45$ eq(1)
And
$B\cos\theta\cos30 = B'\cos45$ eq(2)
Taking ratio of eq(1) and eq(2) we get

$$\frac{B\sin\theta}{B\cos\theta\cos30} = \frac{B'\sin45}{B'\cos45}$$
$$\tan\theta = \cos30$$
$$\tan\theta = \frac{\sqrt{3}}{2} = 0.866$$
$$\theta = \tan^{-1}(0.866)$$

Tangent law

A magnetic needle suspended, at a point where there are two crossed magnetic fields acting at right angles to each other, will come to rest in the direction of the resultant of the two fields.



B_1 and B_2 are two uniform magnetic fields acting at right angles to each other. A magnetic needle placed in these two fields will be subjected to two torques tending to rotate the magnet in opposite directions. The torque τ_1 due to the two equal and opposite parallel forces mB_1 and mB_1 tend to set the magnet parallel to B_1. Similarly the torque τ_2 due to the two equal and opposite parallel forces mB_2 and mB_2 tends to set the magnet parallel to B_2. In a position where the torques balances each other, the magnet comes to rest. Now the magnet makes an angle θ with B_2 as shown in the Fig.
The deflecting torque due to the forces mB_1 and mB_1
$\tau_1 = mB_1 \times NA$
$= mB_1 \times NS\cos\theta$
$= mB_1 \times 2l\cos\theta$
$= 2l\, mB_1\cos\theta$
$\therefore \tau_1 = MB_1\cos\theta$
Similarly the restoring torque due to the forces mB_2 and mB_2
$\tau_2 = mB_2 \times SA$
$= mB_2 \times 2l\sin\theta$
$= 2lm \times B2\sin\theta$
$\tau_2 = MB2\sin\theta$
At equilibrium,
$\tau_1 = \tau_2$
$\therefore MB_1\cos\theta = MB_2\sin\theta$
$\therefore B_1 = B_2\tan\theta$
This is called Tangent law
Invariably, in the applications of tangent law, the restoring magnetic

field B_2 is the horizontal component of Earth's magnetic field B_H.

Solved Numerical

A short bar magnet of magnetic moment 5.25×10^{-2} A m^2 is placed with its axis perpendicular to the Earth's field direction. At what distance from the centre of the magnet on (i) its equatorial line and (ii) its axial line, is the resultant field inclined at 45° with the Earth's field. Magnitude of the Earth's field at the place is 0.42×10^{-4} T.

Solution

From Tangent Law

$$\frac{B}{B_H} = \tan\theta$$

$$B = B_H \tan\theta = 0.42 \times 10^{-4} \times \tan 45^\circ$$

$$B = 0.42 \times 10^{-4}\ \text{T}$$

(i) For the point on the equatorial line

$$B = \frac{\mu_0}{4\pi}\frac{m}{d^3}$$

$$d^3 = \frac{\mu_0}{4\pi}\frac{m}{B}$$

$$d^3 = 10^{-7} \times \frac{5.25 \times 10^{-2}}{0.42 \times 10^{-4}}$$

$$d = 5 \times 10^{-2}\ \text{m}$$

(ii) For the point on the axial line

$$B = \frac{\mu_0}{4\pi}\frac{2m}{d^3}$$

$$d^3 = \frac{\mu_0}{4\pi}\frac{2m}{B}$$

$$d^3 = 10^{-7} \times \frac{2 \times 5.25 \times 10^{-2}}{0.42 \times 10^{-4}}$$

$$d = 6.3 \times 10^{-2}\ \text{m.}$$

Magnetic properties of materials

The study of magnetic properties of materials assumes significance since these properties decide whether the material is suitable for permanent magnets or electromagnets or cores of transformers etc.

Before classifying the materials depending on their magnetic behavior, the following important terms are defined.

(i) Magnetizing field or magnetic intensity

The magnetic field used to magnetize a material is called the magnetizing field. It is denoted by H and its unit is A m^{-1}.

(Note : Since the origin of magnetism is linked to the current, the magnetizing field is usually defined in terms of ampere turn)

(ii) Magnetic permeability

Magnetic permeability is the ability of the material to allow the passage of magnetic lines of force through it. *Relative permeability* μ_r *of a material is defined as the ratio of number of magnetic lines of force per unit area B inside the material to the number of lines of force per unit area in vacuum* B_o *produced by the same magnetizing field.*

∴ Relative permeability $\mu_r = B / B_O$

$$\mu_r = \frac{\mu H}{\mu_0 H} = \frac{\mu}{\mu_0}$$

(since μ_r is the ratio of two identical quantities, it has no unit.)

∴ The magnetic permeability of the medium $\mu = \mu_o\mu_r$ where μ_o is the

permeability of free space.
Magnetic permeability μ of a medium is also defined as the ratio of magnetic induction B inside the medium to the magnetizing field H inside the same medium.

$$\mu = \frac{B}{H}$$

(iii) Intensity of magnetization
Intensity of magnetization represents the extent to which a material has been magnetized under the influence of magnetizing field H. Intensity of magnetization of a magnetic material is defined as the magnetic moment per unit volume of the material.

$$M = \frac{m}{V}$$

Its unit is A m^{-1}.
For a specimen of length 2l, area A and pole strength m,

$$M = \frac{2lm}{2lA}$$

$$M = \frac{m}{A}$$

Hence, intensity of magnetization (M) is also defined as the pole strength per unit area of the cross section of the material.

(iv) Magnetic induction
When a soft iron bar is placed in a uniform magnetizing field H, the magnetic induction inside the specimen B is equal to the sum of the magnetic induction B_o produced in vacuum due to the magnetizing field and the magnetic induction B_m due to the induced magnetization of the specimen.
$B = B_o + B_m$
But $B_o = \mu_o H$ and $B_m = \mu_o M$
$B = \mu_o H + \mu_o M$
$\therefore B = \mu_o (H + M)$

$$H = \frac{B}{\mu o} - M$$

where **H** has the same dimensions as **M** and is measured in units of A m^{-1}.
Thus, the total magnetic field **B**

(v) Magnetic susceptibility
Magnetic susceptibility χ_m is a property which determines how easily and how strongly a specimen can be magnetized.
Susceptibility of a magnetic material is defined as the ratio of intensity of magnetization induced in the material to the magnetizing field H in which the material is placed.
Thus

$$\chi_m = \frac{M}{H}$$

Since *I* and *H* are of the same dimensions, χ_m has no unit and is dimensionless.

Relation between χ_m and μ_r

$$\chi_m = \frac{M}{H}$$

$$M = \chi_m H$$

We know $B = \mu_o (H + M)$

$$B = \mu_o (H + \chi_m H)$$

$$B = \mu_o H (1 + \chi_m)$$

If μ is the permeability, we know that $B = \mu H$.

$$\therefore \mu H = \mu_o H (1 + \chi_m)$$

$$\frac{\mu}{\mu_0} = (1 + \chi_m)$$

$$\mu_r = 1 + \chi_m$$

Solved Numerical

A bar magnet of mass 90 g has magnetic moment 3 A m^2. If the intensity of magnetization of the magnet is 2.7 × 10^5 A m^{-1}, find the density of the material of the magnet.

<u>Solution</u>

Intensity of magnetization, $M = \frac{m}{V}$

volume V =mass/ ρ

$$M = \frac{m\rho}{mass}$$

$$\rho = \frac{M \times mass}{m} = \frac{2.7 \times 10^5 \times 0.090}{3}$$

$$\rho = 8100 \, \frac{kg}{m^3}$$

Q) A magnetizing field of 50 A m^{-1} produces a magnetic field of induction 0.024 T in a bar of length 8 cm and area of cross section 1.5 cm^2. Calculate (i) the magnetic permeability (ii) the magnetic susceptibility

<u>Solution</u>

Permeability

$$\mu = \frac{B}{H} = \frac{2.4 \times 10^{-2}}{50} = 4.8 \times 10^{-4} Hm^{-1}$$

susceptibility

$$\chi_m = \frac{\mu}{\mu_0} - 1$$

$$\chi_m = \frac{4.8 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 381.16$$

Q) A solenoid has a core of material with relative permeability of 400. The current passing through the wire of solenoid is 2A. If the number of turns per cm are 10, calculate the magnitude of

(a) H (b) B (c) χ_m (d) M

<u>Solution</u>

Here μ_r = 400, I = 2A n = 10 turns/cm = 1000 turns/m

(a) Magnetic intensity H = nI = 1000 × 2 = 2000 Am^{-1}

(b) Magnetic field B = $\mu_o \mu_r H = 4\pi \times 10^{-7} \times 400 \times 2000 = 1.0$ T

(c) Magnetic susceptibility of the core material is

$\chi_m = \mu_r - 1 = 400 - 1 = 399$

(d) Magnetization

M = $\chi_m H$ = 399 × 2000 = 8 ×10^5 A/m

Q) The region inside a current carrying toroidial winding is filled with tungsten of susceptibility 6.8×10^{-5}. What is the percentage increase in the magnetic field in the presence of the material with respect to the magnetic field without it?

Solution:

The magnetic field in the current carrying toroidial winding without tungsten is

$B_O = \mu_O H$

The magnetic field in the same current carrying toroidial winding with tungsten is

B = μH

$$\therefore \frac{B - B_o}{B_o} = \frac{\mu - \mu_o}{\mu_o}$$

But $\mu = \mu_o (1 + \chi_m)$

$$\frac{\mu}{\mu_o} = 1 + \chi_m$$

$$\chi_m = \frac{\mu - \mu_O}{\mu_0}$$

$$\therefore \frac{B - B_o}{B_o} = \chi_m$$

$$\therefore \frac{B - B_o}{B_o} \times 100 = \chi_m \times 100$$

$$\therefore \frac{B - B_o}{B_o} \times 100 = (6.8 \times 10^{-5}) \times 100$$

$$\therefore \frac{B - B_o}{B_o} \times 100 = (6.8 \times 10^{-3})\%$$

Classification of magnetic materials

On the basis of the behavior of materials in a magnetizing field, the materials are generally classified into three categories namely,
(i) Diamagnetic, (ii) Paramagnetic and (iii) Ferromagnetic

(i) Properties of diamagnetic substances

Diamagnetic substances are those in which the net magnetic moment of atoms is zero. The susceptibility has a low negative value.
(For example, for bismuth $\chi_m = -0.00017$).
2. Susceptibility is independent of temperature.
3. The relative permeability is slightly less than one.
4. When placed in a non uniform magnetic field they have a tendency to move away from the field (i.e) from the stronger part to the weaker part of the field. They get magnetized in a direction opposite to the field.
5. When suspended freely in a uniform magnetic field, they set themselves perpendicular to the direction of the magnetic field
Examples : Bi, Sb, Cu, Au, Hg, H_2O, H_2 etc.

(ii) Properties of paramagnetic substances

Paramagnetic substances are those in which each atom or molecule has a net non-zero magnetic moment of its own.
1. Susceptibility has a low positive value.
(For example : χ_m for aluminium is +0.00002).
2. Susceptibility is inversely proportional to absolute temperature. As the temperature increases susceptibility decreases.
3. The relative permeability is greater than one.
4. When placed in a non uniform magnetic field, they have a tendency to move from weaker part to the stronger part of the field. They get magnetized in the direction of the field.
5. When suspended freely in a uniform magnetic field, they set themselves parallel to the direction of magnetic field
Examples : Al, Pt, Cr, O_2, Mn, $CuSO_4$ etc.
Pierre Curie observed the magnetization M of a paramagnetic material is directly proportional to the external magnetic filed B and inversely proportional to its absolute temperature T, called Curie's law

$$M = C\frac{B}{T}$$

Where C = Curie's constant

$$M = C\frac{B}{T}\frac{\mu_0}{\mu_0}$$

$$M = CH\frac{\mu_0}{T}\left(\because H = \frac{B}{\mu_0}\right)$$

$$\frac{M}{H} = \chi_m = C\frac{\mu_0}{T}$$

(iii) Properties of ferromagnetic substances

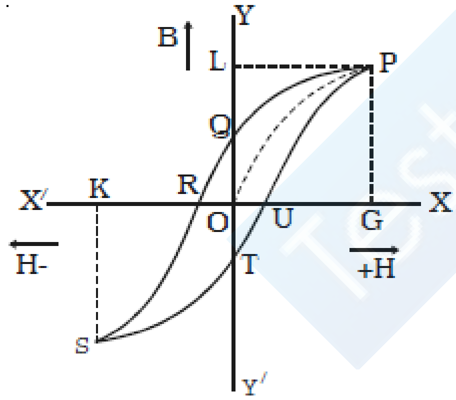
Ferromagnetic substances are those in which each atom or molecule has a strong spontaneous net magnetic moment. These substances exhibit strong paramagnetic properties.
1. The susceptibility and relative permeability are very large.
(For example : μ_r for iron = 200,000)
2. Susceptibility is inversely proportional to the absolute temperature.
As the temperature increases the value of susceptibility decreases. *At a particular temperature, ferromagnetic become paramagnetic. This transition temperature is called Curie temperature.*
The relation between magnetic susceptibility of the substance in the acquired paramagnetic form and temperature is given by

$$\chi_m = \frac{C_1}{T - T_C}$$

C_1 is a constant
For example: Curie temperature of iron is about 1000 K.
3. When suspended freely in uniform magnetic field, they set themselves parallel to the direction of magnetic field.
4. When placed in a non uniform magnetic field, they have a tendency to move from the weaker part to the stronger part of the field. They get strongly magnetized in the direction of the field.
Examples : Fe, Ni, Co and a number of their alloys.

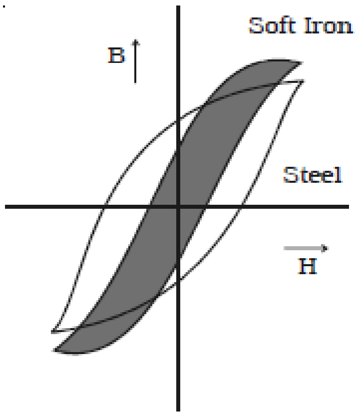
Hysteresis

Consider an iron bar being magnetized slowly by a magnetizing field H whose strength can be changed. It is found that the magnetic induction B inside the material increases with the strength of the magnetizing field and then attains a saturated level. This is depicted by the path OP in the



If the magnetizing field is now decreased slowly, then magnetic induction also decreases but it does not follow the path PO. Instead, when H = 0, B has non zero value equal to OQ. This implies that some magnetism is left in the specimen. *The value of magnetic induction of a substance, when the magnetizing field is reduced to zero, is called residual magnetic induction of the material.* OQ represents the residual magnetism of the material. Now, if we apply the magnetizing field in the reverse direction, the magnetic induction decreases along QR till it becomes zero at R. Thus to reduce the residual magnetism (remnant magnetism) to zero, we have to apply a magnetizing field OR in the opposite direction.
The value of the magnetizing field H which has to be applied to the magnetic material in the reverse direction so as to reduce its residual magnetism to zero is called its coercivity.
When the strength of the magnetizing field H is further increased in the reverse direction, the magnetic induction increases along RS till it acquires saturation at a point S (points P and S are symmetrical). If we now again change the direction of the field, the magnetic induction follows the path STUP. *This closed curve PQRSTUP is called the*

'hysteresis loop' and it represents a cycle of magnetization. The word 'hysteresis' literally means lagging behind. We have seen that magnetic induction B lags behind the magnetizing field H in a cycle of magnetization. *This phenomenon of lagging of magnetic induction behind the magnetizing field is called hysteresis.*
In the process of magnetization of a ferromagnetic substance through a cycle, there is expenditure of energy. The energy spent in magnetizing a specimen is not recoverable and there occurs a loss of energy in the form of heat. This is so because, during a cycle of magnetization, the molecular magnets in the specimen are oriented and reoriented a number of times. This molecular motion results in the production of heat. It has been found that *loss of heat energy per unit volume of the specimen in each cycle of magnetization is equal to the area of the hysteresis loop.* The shape and size of the hysteresis loop is characteristic of each material because of the differences in their retentivity, coercivity, permeability, susceptibility and energy losses etc. By studying hysteresis loops of various materials, one can select suitable materials for different purposes.
At $H = 0$, $B \neq 0$. The value of B at $H = 0$ is called retentivity or remanence.
At $H \neq 0$, $B = 0$. The value of H at $B = 0$ is called coercivity.
Hysteresis loss



In the process of magnetization of a ferromagnetic substance through a cycle, there is expenditure of energy. The energy spent in magnetizing a specimen is not recoverable and there occurs a loss of energy in the form of heat. This is so because, during a cycle of magnetization, the molecular magnets in the specimen are oriented and reoriented a number of times. This molecular motion results in the production of heat. It has been found that *loss of heat energy per unit volume of the specimen in each cycle of magnetization is equal to the area of the hysteresis loop.* The shape and size of the hysteresis loop is characteristic of each material because of the differences in their retentivity, coercivity, permeability, susceptibility and energy losses etc. By studying hysteresis loops of various materials, one can select suitable materials for different purposes.

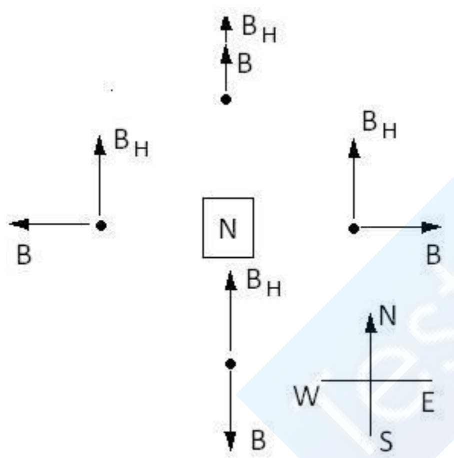
Uses of ferromagnetic materials

(i) Permanent magnets
The ideal material for making permanent magnets should possess high retentivity (residual magnetism) and high coercivity so that the magnetization lasts for a longer time. Examples of such substances are steel and alnico (an alloy of Al, Ni and Co).
(ii) Electromagnets
Material used for making an electromagnet has to undergo cyclic changes. Therefore, the ideal material for making an electromagnet has to be one which has the least hysteresis loss. Moreover, the material should attain high values of magnetic induction B at low values of magnetizing field H. Soft iron is preferred for making electromagnets as it has a thin hysteresis loop[small area, therefore less hysteresis loss] and low retentivity.It attains high values of B at low values of magnetizing field.
(iii) Core of the transformer

A material used for making transformer core and choke is subjected to cyclic changes very rapidly. Also, the material must have a large value of magnetic induction B. Therefore, soft iron that has thin and tall hysteresis loop is preferred. Some alloys with low hysteresis loss are: radio-metals, pern-alloy.
(iv) Magnetic tapes and memory store
Magnetization of a magnet depends not only on the magnetizing field but also on the cycle of magnetization it has undergone. Thus, the value of magnetization of the specimen is a record of the cycles of magnetization it has undergone. Therefore, such a system can act as a device for storing memory. Ferro magnetic materials are used for coating magnetic tapes in a cassette player and for building a memory store in a modern computer. Examples : Ferrites (Fe, Fe_2O, $MnFe_2O_4$ etc.).

Questions
Q) It is observed that the neutral points lie along the axis of a magnet placed on the table. What is the orientation of the magnet with respect to the earth's magnetic field
Ans. North pole of the magnet is towards the south of the earth
Q) A bar magnet is stationary in magnetic meridian. Another similar magnet is kept to it such that the centre lie on their perpendicular bisectors. If the second magnet is free to move, then what type of motion it will have - translator, rotator or both
Ans: Only translator
Q) A short bar magnet placed with its axis making an angle θ with a uniform external field B experiences a torque. What is the magnetic moment of the magnet
Q) Name the parameters needed to completely specify the earth's magnetic field at a point on the earth's surface
Ans: Declination, Dip and Horizontal component of earth's field
Q) What is geomagnetic equator
Ans: The great circle on the earth's surface whose plane is perpendicular to the magnetic axis is called magnetic equator.
Q) What is magnetic meridian
Ans: A vertical plane passing through the magnetic axis of earth is called magnetic meridian
Q) Name the physical quantity which is measured in Wb A^{-1}
Ans: The ratio of the magnetic induction and the magnetic moment is measured in Wb A^{-1}
Q) Name one property of magnetic material used for making permanent magnet
Ans: High coercivity
Q) The ratio of the horizontal component to the resultant magnetic field of earth at a given place is $(1/\sqrt{2})$. What is the angle of dip at that place
Ans : $cos\theta = \frac{B_H}{B} = \frac{1}{\sqrt{2}}$
$\theta = 45^O$
Q) Why does a paramagnetic sample display greater magnetization (for same magnetizing field) when cooled
Ans: The tendency to disrupt the alignment of dipoles with the magnetizing field arising from random thermal motion is reduced at lower temperatures. So, as the paramagnetic substance is cooled, its atomic dipoles tends to get aligned with the magnetizing field. Thus, the paramagnetic substance display a greater magnetization when cooled
Q) What is SI unit of magnetic permeability?
Ans: T m A^{-1}
Q) Why do magnetic lines of force prefer to pass through iron than air
Ans: Permeability of soft iron is greater than that of air
Q) What is the SI unit of susceptibility
Ans: It has no unit
Q) Identify a substance, which has negative magnetic susceptibility.
Ans: Diamagnetic substance. Magnetic susceptibility is positive for both para and ferromagnetic substance
Q) What is the net magnetic moment of an atom of a diamagnetic material

Ans: Zero
Q) What is the dimensional formula of magnetic flux
Ans: [$ML^2T^{-2}A^{-1}$]
Q) An iron nail is attracted by a magnet. What is the source of kinetic energy
Ans: It is the magnetic field energy which is partly converted into kinetic energy
Q) A bar magnet is cut into two equal pieces transverse to its length. What happens to its dipole moment
Ans: The magnetic moment will be halved because length will be halved
Q) What is magnet
Ans: A magnet is an arrangement of two equal and opposite magnetic poles separated by a certain distance. It has attractive and directive properties
Q) What is the SI unit of magnetic moment of a dipole
Ans: Am^2 or JT^{-1}
Q) What is Hysteresis?
Ans: Hysteresis is defined as the lagging of the magnetic induction B behind the corresponding magnetic field H
Q) Define angle of magnetic dip
Ans: It is the angle made by the direction of earth's total magnetic field with the horizontal component of the earth's magnetic field at magnetic poles
Q) What is the effect on the magnetization of diamagnetic substance when it is cooled
Ans: The magnetization of a diamagnetic substance is independent of temperature
Q) A magnet is held vertically on a horizontal plane. How many neutral points are there in the horizontal plane



Ans.) The magnetic field due to the magnet and the magnetic field of earth are shown at four different points a, b, c and d. Clearly, the two fields cancel only at the point a. So, a is the neutral point.

Q) In the stirrup of a vibration magnetometer are placed two magnets one above the other with their axes parallel. When will their time period be maximum/minimum
Ans: The time period will be maximum when opposite poles are together

$$T_{max} = 2\pi\sqrt{\frac{I_1 + I_2}{(m_1 - m_2)B_H}}$$

The time period will be minimum when like poles are together

$$T_{max} = 2\pi\sqrt{\frac{I_1 + I_2}{(m_1 + m_2)B_H}}$$

Q) Two substances A and B have their relative permeability slightly greater and less than unity respectively. What do you conclude about A and B
Ans: $\chi_m = \mu_r - 1$
Relative permeability of A is slightly greater than 1. So χ_m is small and positive. So, substance is paramagnetic.
Relative permeability of B is slightly less 1
So χ_m is small and negative. Clearly, substance is diamagnetic
Q) How does the knowledge of declination at a place help in navigation?

Ans: Declination at place gives us the angle between the geographic and the magnetic meridians. So, the knowledge of declination shall help in steering the ship in the required direction so as to reach the destination

Q) Two identical-looking iron bars A and B given, one of which is definitely known to be magnetized [We don't know which one]. How would one ascertain whether or not both are magnetized? If only one is magnetized, how does one ascertain which one? [Use nothing but the two bars A and B]

Ans: Try to bring different ends of the magnets closer. A repulsive force in some situation establishes that both are magnetized. If it is always attractive, then one of them is not magnetized. To see which one, pick up one say A and lower one of its ends: first one of the ends of their other say b, and then on the middle part of B. A experiences no force, and then B is magnetized. If you do not notice any change from end to middle point Of B, then A is magnetized.

Q) A magnetized needle in a uniform magnetic field experiences a torque but no net force. An iron nail near a bar magnet, however, experiences a force of attraction in addition to a torque. Why?

Ans: In the case of uniform magnetic field, the forces experienced by the needle are equal in magnitude, opposite in direction and have different lines of action. So, net force is zero. But torque is not zero

The iron nail experiences a non-uniform magnetic field due to the bar magnet. The induced magnetic moment in the nail, therefore, experiences both force and torque. The net force is attractive because the induced (say) south pole in the nail is closer to the north pole of the magnet than the induced north pole

Q) Why two magnetic lines of force due to a bar magnet do not cross each other?

Ans: If two magnetic lines of force cross at a point, then this would mean that there are two directions of magnetic field at the point of crossing. This is physically absurd. Thus, two magnetic lines of force cannot cross each other

Q) What is the basic use of hysteresis curve?

Ans: Hysteresis loop gives useful information about the different properties, of materials, such as coercivity, retentivity, energy loss. This information helps us in the suitable selection of materials for different purposes.

Q) Does the magnetization of paramagnetic salt depend on temperature? Justify your answer

Ans: The atoms of a paramagnetic substance posses small magnetic dipole moments. But these atomic dipoles are oriented in a random manner. In the presence of the external magnetic field, these dipoles tend to align in the direction of the field. But the tendency for alignment is hindered by thermal agitation. So, the magnetization of paramagnetic salt decreases with increase of temperature.

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