Position of points in two dimensions is represented in vector form

$$\vec{r}_1 = x_1\hat{\imath} + y_1\hat{\jmath} \;\; and \;\; \vec{r}_2 = x_2\hat{\imath} + y_2\hat{\jmath}$$

If particle moves from r_1 position to r_2 position then

Displacement is final position – initial position

$$\vec{s} = \vec{r}_2 - \vec{r}_1$$

$$\vec{s} = (x_2\hat{\imath} + y_2\hat{\jmath}) - (x_1\hat{\imath} + y_1\hat{\jmath} \;\;)$$

$$\vec{s} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath}$$

If $x_2 - x_1 = dx$ and $y_2 - y_1 = dy$

$$\vec{s} = dx\hat{\imath} + dy\hat{\jmath}$$

Now

$$\vec{v} = \frac{d\vec{s}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$$

$dx/dt = v_x$ is velocity of object along x-axis and $dy/dt = v_y$ is velocity along y-axis

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$

Thus motion in two dimension is resultant of motion in two independent component motions taking place simultaneously in mutually perpendicular directions.

Magnitude of velocity

$$v = \sqrt{v_x^2 + v_y^2}$$

Angle between velocity and x-axis is

$$tan\theta = \frac{v_y}{v_x}$$

$$\vec{a} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath}$$

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$$

Magnitude of acceleration

$$a = \sqrt{a_x^2 + a_y^2}$$

Equation of motion in vector form

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{r} = \vec{r}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$average\ velocity = \frac{\vec{u} + \vec{v}}{2}$$

$$v^2 - u^2 = 2\vec{a} \cdot (\vec{r} - \vec{r}_0)$$

Solved Numerical

Q1) A particle starts its motion at time t = 0 from the origin with velocity 10**j** m/s and moves in X-Y plane with constant acceleration 8**i**+2**j**

(a) At what time its x-coordinate becomes 16m? And at that time what will be its y co-ordinate?

(b) What will be the speed of this particle at this time?

Solution:

From equation of motion

$$\vec{r} = \vec{r}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$x\hat{\imath} + y\hat{\jmath} = (10\hat{\jmath})t + \frac{1}{2}(8\hat{\imath} + 2\hat{\jmath})t^2$$

$$x\hat{\imath} + y\hat{\jmath} = (4t^2)\hat{\imath} + (10t + t^2)\hat{\jmath}$$

Thus

$$x = 4t^2$$

When x = 16

$16 = 4t^2$, t = 2 sec

Now $y = 10t+t^2$ at t = 2 sec

Y = 20+4 = 240 m

b)

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{v} = 10\hat{j} + (8\hat{\imath} + 2\hat{j})t$$

t =2 sec

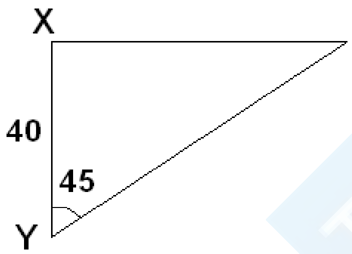
$$\vec{v} = 10\hat{j} + (8\hat{\imath} + 2\hat{j}) \times 2$$

$$\vec{v} = 16\hat{\imath} + 14\hat{j}$$

$$\overrightarrow{|v|} = \sqrt{16^2 + 14^2} = 21.26\, sec$$

Q2) A bomber plane moves due east at 100km/hr over a town X at a certain time, Six minutes later an interceptor plans sets off from station Y which is 40km due south of X and flies north east. If both maintain their courses, find the velocity with which interceptor must fly in order to take over the bomber

Solution:



Let velocity of interceptor speed be V . Now components of velocity along North direction is Vcos45 and along East direction is Vsin45

To intercept the bomber , interceptor has to travel a distance of 40 km in t seconds along North direction Thus 40 = Vcos45t

Vt= 40√2 ---- eq(1)

Now total distance travelled by bomber = 100t + distance travelled in 6 minutes before interceptor takeoff

Distance travelled by bomber along East direction = 100t +10 km

Thus distance travelled by interceptor

100t + 10 = Vsin45t

100t +10 = Vt / √2

From eq(1)

100t +10 =40

t = 3/10 Hr

Substituting above value of t in equation (1) we get

$$V = \frac{40\sqrt{2} \times 10}{3} = 188.56\ km/hr$$

Q3) A motor boat set out at 11a.m. from a position -6**i**-2**j** relative to a marker buoy and travels at steady speed of magnitude √53 on direct course to intercept a ship. This ship maintains a steady velocity vector 3**i**+4**j** and at 12 noon is at position 3**i**-4**j** from the buoy. Find (a) the velocity vector of the motorboat (b) the time of interception and (c) the position vector of point of interception from the buoy, if distances are measured in kilometers and speeds in km/hr

Solution

Let xi+vj be the velocity vector of the motor boat

$X^2 + y^2 = 53$ ------eq(1)

Position vector of motor boat after time t = -6**i**-2**j** + (x**i**+y**j**)t

Since position of ship is given 12 noon to find position of ship at time t, time for ship = (t-1)

Position vector of ship after time t = 3**i**-**j**+(3**i**+4**j**) (t-1)

The time of interception is given by

-6**i**-2**j**+(x**i**+y**j**)t = 3**i**-**j** +(3**i**+4**j**) (t-1)

∴-6+xt = 3t

∴xt = 6 +3t -------eq(2)

And -2yt = 4t-5

∴yt = 4t-3 ------eq(3)

From equation (1), (2) and (3)

$(6+3t)^2 + (4t-3)^2 = 53t^2$

$\therefore 36+9t^2+36t +16t^2-24t+9 = 53t^2$

∴28t-12t-45 = 0

∴(2t-3) (14t +15) = 0

t = (3/2) hr

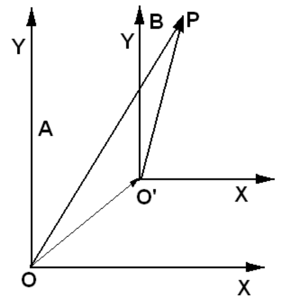
Time of interception = 12.30 pm

Solving equation(2) and (3) x = 7 and y = 2

Velocity vector of motor boat = 7**i**+2**j**

Point of interception = -6**i**-2**j** + (7**i** +2**j**) (3/2) = 9/2 **i** +**j**

Relative velocity



Two frame of reference A and B as shown in figure moving with uniform velocity with respect to each other. Such frame of references are called inertial frame of reference. Suppose two observers, one in from A and one from B study the motion of particle P.

Let the position vectors of particle P at some instant of time with respect to the origin O of frame A be

$$\vec{r}_{P,A} = \overrightarrow{OP}$$

And that with respect to the origin O′ of frame B be

$$\vec{r}_{P,B} = \overrightarrow{O'P}$$

The position vector of O′ w.r.t O is

$$\vec{r}_{B,A} = \overrightarrow{OO'}$$

From figure it is clear

$$\overrightarrow{OP} = \overrightarrow{OO'} + \overrightarrow{O'P} = \overrightarrow{O'P} + \overrightarrow{OO'}$$

$$\therefore \vec{r}_{P,A} = \vec{r}_{P,B} + \vec{r}_{B,A}$$

Differentiating above equation with respect to time we get

$$\frac{d}{dt}(\vec{r}_{P,A}) = \frac{d}{dt}(\vec{r}_{P,B}) + \frac{d}{dt}(\vec{r}_{B,A})$$

$$\therefore \vec{v}_{P,A} = \vec{v}_{P,B} + \vec{v}_{B,A}$$

Here $\mathbf{V}_{P,A}$ is the velocity of the particle w.r.t frame of reference A, $\mathbf{V}_{P,B}$ is the velocity of the particle w.r.t. reference frame B and $V_{B,A}$ is the velocity of frame of reference B with respect to frame A

Suppose velocities of two particles A and B are respectively $\mathbf{V}_A$ and $\mathbf{V}_B$ relative to frame of reference then velocity ($\mathbf{V}_{AB}$) of A with respect to B is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

And velocity $\mathbf{V}_{BA}$ of B relative to A is

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Thus

$$\vec{v}_{AB} = -\vec{v}_{BA}$$

And

$$|\vec{v}_{AB}| = |\vec{v}_{BA}|$$

$$|\vec{v}_{AB}| = \sqrt{V_A^2 + V_B^2 - 2V_AV_B\cos\theta}$$

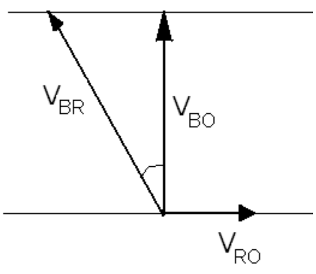
Also angle α made by the relative velocity with V_A is given by

$$\tan\alpha = \frac{V_B\sin\theta}{V_A - V_B\cos\theta}$$

Solved Numerical

Q4) A boat can move in river water with speed of 8 km/h. This boat has to reach to a place from one bank of the river to a place which is in perpendicular direction on the other bank of the river. Then (i) in which direction should the boat has to be moved (ii) If the width of the river is 600m, then what will be the time taken by the boat to cross the river? The river flows with velocity 4 km/h

Solution



Suppose the river is flowing in positive X direction as shown in figure . To reach to a place in the perpendicular direction on the other bank, the boat has to move in the direction making angle θ with Y direction as shown in figure. . This angle should be such that the velocity of the boat relative to the opposite bank is in the direction perpendicular to the bank . Let V_{BR} = Velocity of boat with respect to river

V_{BO} = Velocity of Boat with respect to Observer on bank

V_{RO} = velocity of river with respect to observer on bank

$$\vec{V}_{BO} = \vec{V}_{BR} + \vec{V}_{RO}$$

In vector form

$$V_{BO}\hat{j} = -V_{BR}sin\theta\hat{\imath} + V_{BR}cos\theta\hat{j} + V_{RO}\hat{\imath}$$

$$V_{BO}\hat{j} = -8sin\theta\hat{\imath} + 8cos\theta\hat{j} + 4\hat{\imath}$$

Thus considering only x components

$$0 = -8sin\theta + 4$$

$$\Rightarrow \theta = 30^{O}$$

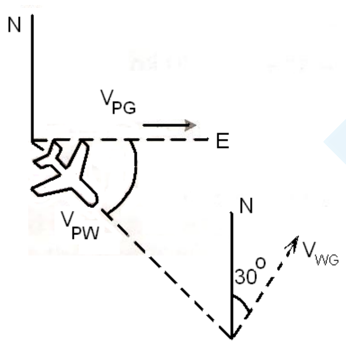
$$V_{BO} = 8cos\theta \ , \qquad V_{BO} = 8cos30$$

$$V_{BO} = 8\frac{\sqrt{3}}{2} = 4\sqrt{3} = 6.93\ km/h$$

Time taken to cross river

$$t = \frac{distance}{velocity} = \frac{0.6}{6.93} = 0.0865\ Hr = 5.2\ minutes$$

Q5) In figure a plane moves due east while pilot points the plane somewhat south of east, towards a steady wind that blows to the northeast. The plane has velocity V_{PW} relative to the wind, with an airspeed of 200km/hr directed at an angle θ south of east. The wind has velocity relative to the ground, with a speed $40\sqrt{3}$ km/hr directed 30^{O} east of north. What is the magnitude of the velocity of plane V_{PG} relative to the ground, and what is the value of θ



Solution

$$\vec{V}_{PG} = \vec{V}_{PW} + \vec{V}_{WG}$$

$$V_{PG}\hat{\imath} = V_{PW}cos\theta\hat{\imath} - V_{PW}sin\theta\vec{j} + V_{WG}sin30\hat{\imath} + V_{WG}cos30\hat{j}$$

$$V_{PG}\hat{\imath} = V_{PW}cos\theta\hat{\imath} + V_{WG}sin30\hat{\imath} + V_{WG}cos30\hat{j} - V_{PW}sin\theta\hat{j}$$

Comparing y co-ordinates

$$0 = V_{WG}cos30 - V_{PW}sin\theta$$

$$0 = 40\sqrt{3}\frac{\sqrt{3}}{2} - 200sin\theta$$

$$sin\theta = \frac{60}{200} = \frac{3}{10}$$

⇒ θ =17.45°

Comparing x coordinates

$$V_{PG}\hat{\imath} = V_{PW}cos\theta\hat{\imath} + V_{WG}sin30\hat{\imath}$$

$$V_{PG} = 200cos17.45 + 40\sqrt{3}sin30$$

$$V_{PG} = 200cos17.45 + 40\sqrt{3}sin30 = 225\ km/hr$$

Projectile motion

A projectile is a particle, which is given an initial velocity, and then moves under the action of its weight alone.

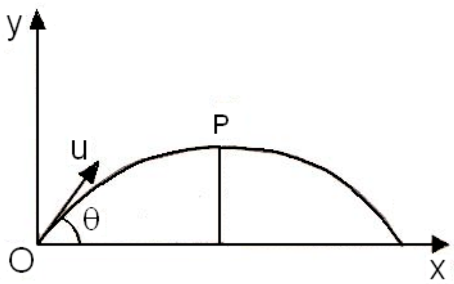
When object moves at constant horizontal velocity and constant vertical downward acceleration, such a two dimension motion is called projectile.

The projectile motion can be treated as the resultant motion of two independent component motion taking place simultaneously in mutually perpendicular directions. One component is along the horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to gravitational force.

Important terms used in projectile motion

When a particle is projected into air, the angle that the direction of projection makes with horizontal plane through the point of projection is called the angle of projection, the path, which the particle describes, is called the trajectory, the distance between the point of projection and the point where the path meets any plane draws through the point of projection is its range, the time that elapses in air is called as time of flight and the maximum distance above the plane during its motion is called as maximum height attained by the projectile

Analytical treatment of projectile motion



Consider a particle projected with a velocity u of an angle θ with the horizontal earth's surface. If the earth did not attract a particle to itself, the particle would describe a straight line, on account of attraction of earth, however, the particle describes a curve path

Let us take origin at the point of projection and x-axis along the surface of earth and perpendicular to it respectively shown in figure

Here gravitational force is the force acting on the object downwards with constant acceleration of g downwards. There if no force along horizontal direction hence acceleration along horizontal direction is zero

Motion in x –direction

Motion in x-direction with uniform velocity

At t = 0, X_0 = 0 and u_x = ucosθ

Position after time t , x = x_0 + u_x t

X = (ucosθ) t -----eq(1)

Velocity at t , V_x = u_x

V_x= ucosθ ------eq(2)

Motion in y-direction:

Motion in y-direction is motion with uniform acceleration

When t = 0, Y_0 = 0, u_y = usinθ and a_y = -g

After time 't', V_y = y_y + a_y t

V_y = usinθ – gt -----eq(3)

$$Y = Y_0 + u_y t + \frac{1}{2} a_y t^2 \quad ---eq(4)$$

Also,

$$V_y^2 = u_y^2 + 2a_y t^2$$

$$V_y^2 = u^2 sin^2\theta - 2gy \quad ----eq(5)$$

Time of flight(T) :

Time of flight is the time duration which particle moves from O to O'.

i.e t = T, Y = 0

From equation (4)

$$0 = usin\theta T - \frac{1}{2}gT^2$$

$$T = \frac{2usin\theta}{g} \qquad ----eq(6)$$

Range of projectile (R):

Range of projectile distance travelled in time T,

i.e. R = V_x × T

R = ucosθT

$$R = ucos\theta\frac{2usin\theta}{g}$$

Since 2sinθcosθ = sin2θ

$$R = \frac{u^2sin2\theta}{g} \qquad ----eq(7)$$

Maximum height reached (H):

At the time particle reaches its maximum height velocity of particle becomes parallel to horizontal direction i.e. V_y = 0, when Y = H

From equation (5)

$$0 = u^2sin^2\theta - 2gH$$

$$H = \frac{u^2sin^2\theta}{2g}$$

Equation of trajectory:

The path traced by a particle in motion is called trajectory and it can be known by knowing the relation between X and Y

From equation (1) and (4) eliminating time t we get

$$Y = Xtan\theta - \frac{1}{2}\frac{gX^2}{u^2}sec^2\theta$$

This is trajectory of path and is equation of parabola. So we can say the path of particle is parabolic

Velocity and direction of motion after a given time:

After time 't'

V_y = $u\cos\theta$ and V_y = $u\sin\theta - gt$

Hence resultant velocity

$$V = \sqrt{V_X^2 + V_Y^2}$$

$$V = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

If direction of motion makes an angle α with horizontal

$$\tan\alpha = \frac{V_y}{V_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

$$\alpha = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$$

Velocity and direction of motion at a given height

At height 'h' , V_x = $u\cos\theta$

And

$$V_y = \sqrt{u^2\sin^2\theta - 2gh}$$

Resultant velocity

$$V = \sqrt{V_x^2 + V_y^2}$$

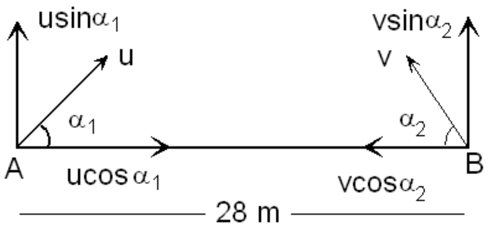
$$V = \sqrt{u^2 - 2gh}$$

Note that this is the velocity that a particle would have at height h if it is projected vertically from ground with u

Solved Numerical

Q6) Two particles are projected at the same instant from point A and B on the same horizontal level where AB = 28 m, the motion taking place in a vertical plane through AB. The particle from A has an initial velocity of39 m/s at an angle $\sin^{-1}$ (5/13) with AB and the particle from B has an initial velocity 25 m/s at an angle $\sin^{-1}$(3/5) with BA. Show that the particle would collide in mid air and find when and where the impact occurs

Solution



If both the particles have same initial vertical velocity v then only particles will collide so that for collision both particles must be at same height

Vertical velocity of particle at A = $u\sin\alpha_1$ =39(5/13) =15 m/s

Vertical velocity of particle at B = $v\sin\alpha_2$ = 25(3/5) = 15 m/s

Since both particles have same vertical velocity they will collide

Since both the particles have travelled in all horizontal distance of 28 m

Therefore 28 = $[u\cos\alpha_1 + v\cos\alpha_2]t$

Now $\sin\alpha_1$ = 5/13 thus $\cos\alpha_1$ = (12/13)

$\sin\alpha_2$ = 3/5 thus $\cos\alpha_2$ = 4/5

Substituting values in equation (1) we get

28 = [39(12/13) + 25(4/5)] t

28 = (56) t

t = 0.5 sec. Particles will collide after 0.5 sec

Now Horizontal distance travelled by particle from A =[$u\cos\alpha_1$]t =[39 (12/13)]0.5= 18 m

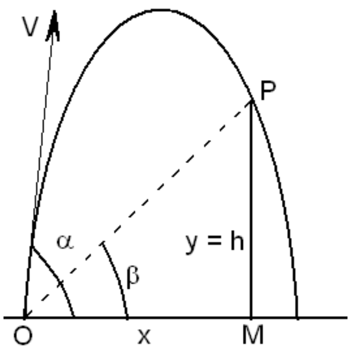
And vertical distance travelled =[$u\sin\alpha_1$]t – $(1/2)gt^2$

=[39(5/13)](0.5) – [0.5(9.8)$(0.5)^2$] = 6.275 m

Hence particle will collide at height of 6.275 m and from 18 m from point A

Q7) A particle is projected with velocity v from a point at ground level. Show that it cannot clear a wall of height h at a distance x from the point of projection if

$v^2 \geq g\left(h + \sqrt{x^2 + h^2}\right)$



Solution:

From the equation for path of projectile

$$y = h = x\tan\alpha - \frac{gx^2}{2v^2}\frac{1}{\cos^2\alpha}$$

$$h = x\tan\alpha - \frac{gx^2}{2v^2}(1 + \tan^2\alpha)$$

This is a quadratic equation in xtanα which is real. Therefore the discriminate cannot be negative.

$$\frac{gx^2}{2v^2}tan^2\alpha - xtan\alpha + \frac{gx^2}{2v^2} + h = 0$$

Here

$$a = \frac{g^2}{2v^2}, \qquad b = 1, \qquad c = \frac{gx^2}{2v^2} + h$$

Now Discriminate D = b^2-4ac ≥ 0

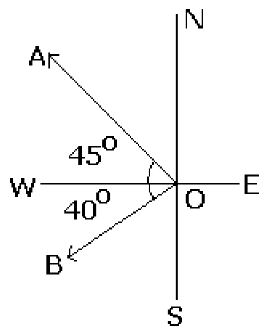
$$1 - 4\left(\frac{g}{2v^2}\right)\left(\frac{gx^2 + 2v^2h}{2v^2}\right) \geq 0$$

$$1 \geq 4\left(\frac{g}{2v^2}\right)\left(\frac{gx^2 + 2v^2h}{2v^2}\right)$$

$$v^4 \geq g^2x^2 + 2v^2hg$$

$$v^2 \geq g\left(h + \sqrt{x^2 + h^2}\right)$$

Q8) Two ships leave a port at the same time. The first steams north-west at 32 km/hr and the second 40^O south of west at 24 km/hr. After what time will they be 160 km apart?



Solution

As show in figure ship goes with 32 km/hr and ship goes with 24 km/hr

We will find relative velocity of Ship A with respect to B so we will get at what speed ships are going away say V_R

$$\vec{V}_R = \vec{V}_A - \vec{V}_B$$

$$V_R^2 = V_A^2 + V_B^2 - 2V_AV_BCos85$$

$$V_R = \sqrt{1024 + 576 - 2(32)(24)(0.0872)}$$

$$V_R = 38.3\frac{km}{hr}$$

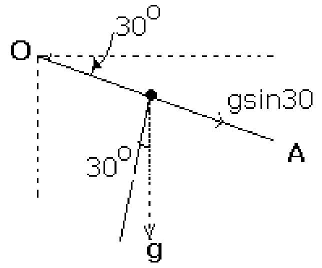
Let t be the time for ships to become 160km apart

38.3×t=160 ∴ t= 4.18 hr

Q9) A object slides off a roof inclined at an angle of 30^O to the horizontal for 2.45m. How far horizontally does it travel in reaching the ground 24.5 m below

Solution

As shown in figure component of gravitational acceleration along the inclined roof is, a=gsinθ = 9.8×(1/2) = 4.9 m/s^2



v^2 = u^2 + 2as

v^2 = 0^2 + 2×4.9×2.45 ⇒ v = 4.9m/s

Now horizontal component of velocity which causes horizontal displacement.

$$V_h = Vcos30 = 4.9 \times \frac{\sqrt{3}}{2}$$

Vertical component $V_v = Vsin30 = 4.9 \times \frac{1}{2} = 2.45\, m/s$

Vertical displacement $h = ut + \frac{1}{2}at^2$

$$24.5 = 2.45 \times t + \frac{1}{2} \times 9.8 \times t^2$$

2t^2+t-10 = 0

Roots are t = 2.5s and -2s

Neglecting negative time

$$Horizontal\ displacement = 2.5 \times 4.9 \times \frac{\sqrt{3}}{2} = 10.6\, m$$

Q10) Two seconds after the projection, a projectile is moving in a direction at 30° to the horizontal, after one more second, it is moving horizontally. Determine the magnitude and direction of the initial velocity.

Solution

Given that in two second angle changed to zero from 30°.

Let y component of velocity at angle 30° by v'_y and final velocity = zero.

0 = V'_y-gt

V'_y = g

Now if V' is the velocity at 30°, then V'_y=V'sin30

∴ g= V' (1/2) or V' = 2g

And x component which remains unchanged = vcos30

$$V_x = 2g \times \frac{\sqrt{3}}{2} = g\sqrt{3}$$

Now in 3 seconds after projection velocity along y axis becomes zero

0= V_y – gt but x component remains unchanged

$V_y = 3g$

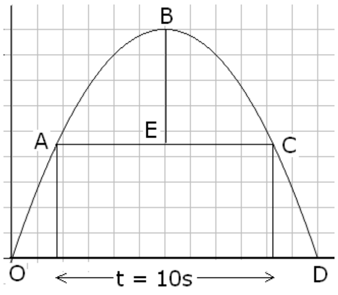
$$V = V_x^2 + V_y^2$$

$$V^2 = \left(g\sqrt{3}\right)^2 + (3g)^2 = 2g\sqrt{3}$$

Angle $tan\theta = \frac{V_y}{V_x} = \frac{3g\sqrt{3}}{g\sqrt{3}} = \sqrt{3}$ θ=60°

Q11) A projectile crosses half its maximum height at a certain instant of time and again 10s later. Calculate the maximum height. If the angle of projection was 30°, calculate the maximum range of the projectile as well as the horizontal distance it travelled in above 10 s.

Solution



Time taken by projectile to travel from point B to C is 5 sec.

And during this time it travels H/2 distance show by BE. Since point B is highest point no vertical component of velocity hence it is a free fall thus

$$\frac{H}{2} = \frac{1}{2} \times 9.8 \times t^2 \quad \therefore \text{H = 245 m}$$

Angle of projection 30° given, from the formula for height

$$H = \frac{u^2 sin^2\theta}{2g}$$

$$245 = \frac{u^2 sin^2 30}{2g} = \frac{u^2}{8g}$$

u = 138.59

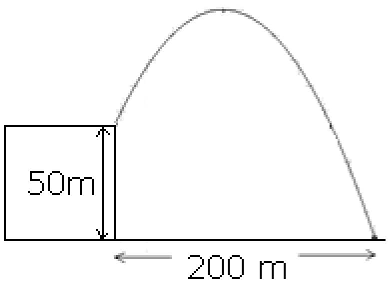
From the formula for range

$$R = \frac{u^2 sin2\theta}{2g} = 1697.4$$

In 10s horizontal distance covered by the particle = ucos30 ×10 ≈1200 m

Q12) A projectile is fired into the air from the edge of a 50-m high cliff at an angle of 30 deg above the horizontal. The projectile hits a target 200 m away from the base of the cliff. What is the initial speed of the projectile, *v*?

Solution



Let v be the velocity of projectile

Velocity along x-axis, V_x = vcosθ

Let t be the time of flight

Displacement along x-axis X, 200= (vcosθ) t

$$t = \frac{200}{vcos\theta} --- eq(1)$$

Velocity along Y-axis vsinθ

Displacement along Y-axis $50 = -(vsin\theta)t + \frac{1}{2}gt^2 - - - eq(2)$

Substituting value of t from eq(1) in eq(2) we get

$$50 = -(vsin\theta)\frac{200}{vcos\theta} + \frac{1}{2}g\left(\frac{200}{vcos\theta}\right)^2$$

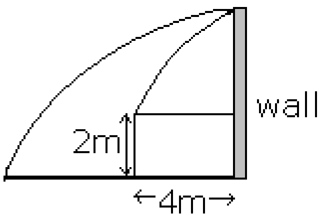
$$50 = -(tan\theta)200 + \frac{1}{2}9.8\left(\frac{200}{vcos\theta}\right)^2$$

$$50 + (tan30)200 = 4.9\left(\frac{200}{vcos30}\right)^2$$

$$v = \sqrt{\frac{4.9 \times 200^2}{cos^2 30 \times [50 + 200tan30]}}$$

V= 39.74 m/s

Q13) A perfectly elastic ball is thrown on wall and bounces back over the head of the thrower, as shown in the figure. When it leaves the thrower's hand, the ball is 2m above the ground and 4m from the wall, and has velocity along x-axis and y-axis is same and is 10m/sec. How far behind the thrower does the ball hit the ground? [assume g = 10 m/s²]



Solution

Displacement along x axis, 4 = 10(t), ∴ t= 0.4sec

Displacement along y-axis

$$y = 10(0.4) - \frac{1}{2}10(0.4)^2 = 3.2\,m$$

Thus ball struck the wall at height 2+3.2 = 5.2 m

Velocity along y-axis of ball when it struck the wall (use formula v=u+at) = 10-10(0.4) = 6m/s

When ball rebounds, direction of y component of velocity remains unchanged. But x component get's reversed.

Component of velocity after bounce are v_x = -10m/s and v_y = 6m/s

Angle with which it rebounds tanθ = 6/10 = 0.6, θ =35°,

With negative x-axis

Now H = 5.2, Time taken to travel H

5.2 = - 6t + ½ ×10×t²

Root of equation t = 1.78 sec.

Distance travelled along x-axis = 1.78× 10 = 17.8 m

Ball falls behind person = 17.8 – 4 = 13.8 m

Q14) An aeroplane has to go to from a point O to another point A, 500 km away 30° east of north. A wind is blowing due north at a speed of 20 m/s. the air –speed of the plane is 150 m/s. [HC Verma, problem 49]

Solution

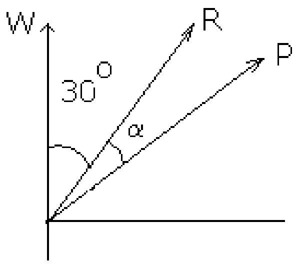


fig a

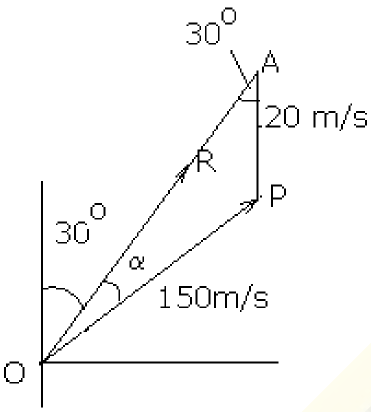


fig b

In fig(a) Vector W represents direction of velocity of wind. Vector P represents direction of plane and vector R represents position where plane has to go.

In fig b. Wind velocity vector W is added to velocity vector of plane P. and R is resultant velocity. From the geometry of fig(b) ∠OAP = 30°.

By sine formula

$$\frac{150}{sin30} = \frac{20}{sin\alpha}$$

Sinα = 0.066 or α = 3°48′

Thus angle between Wind and Plane of velocity is 30+3° 48′ = 33°48′

Thus $R^2 = W^2 + P^2 + 2WP\cos 33°48'$

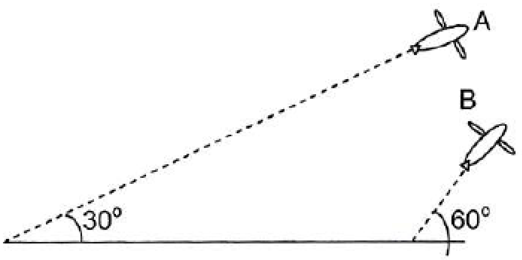
$R^2 = 20^2 + 150^2 + 2\times20\times150\ (0.8310) = 27886$

R = 167 m/s

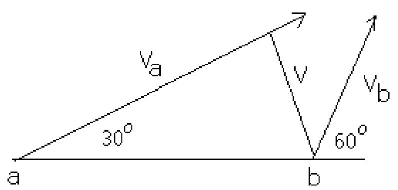
Distance is 500km time taken t

$$t = \frac{500,000}{167} = 2994\ s$$

Q15) Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}\ ms^{-1}$. At time t = 0 s, an observer in A finds B at distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If a t = t_0, A just escapes being hit by B, t_0 in seconds is



Solution:



From the geometry of figure V = $V_a$tan30

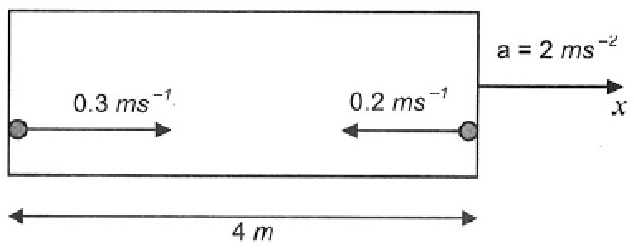
$$V = 100\sqrt{3} \times \frac{1}{\sqrt{3}} = 100m/s$$

Distance for V is 500 m

t_o = 500/100 = 5sec

Q16) A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along +

x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in −x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is



Solution:

Let A have velocity 0.3m/s and B have velocity 0.2 m/s.

Ball A will face acceleration opposite to its motion as rocket is moving with positive acceleration Thus A ball will travel first in positive direction then negative direction. Ball B will travel up to the wall opposite for collision

For ball B direction of acceleration is in the direction of velocity i.e. 2m/s^2

$$4 = 0.2 \times t + \frac{1}{2} \times 2 \times t^2$$

Root is t = 1.99 ≈ 2s

Thus balls will collide after 2sec

Q17) A coin is drop in the elevator from height of 2m. Elevator accelerating down with certain acceleration. Coin takes 0.8 sec. find acceleration of elevator

Solution:

Since elevator is going down with positive acceleration.

Resultant acceleration of coin = g-a,

$$2 = 0 + \frac{1}{2}(g - a)t^2$$

$$2 = 0 + \frac{1}{2}(9.8 - a)(0.8)^2$$

a = 3.55 m/s^2

Q18) A particle starts its motion at time t = 0 from the origin with velocity 10**j** m/s and moves in X-Y plane with constant acceleration 8**i**+2**j**

(a) At what time its x-coordinate becomes 16m? And at that time what will be its y co-ordinate?

(b) What will be the speed of this particle at this time?

Solution:

From equation of motion

$$\vec{r} = \vec{r}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$x\hat{\imath} + y\hat{\jmath} = (10\hat{\jmath})t + \frac{1}{2}(8\hat{\imath} + 2\hat{\jmath})t^2$$

$$x\hat{\imath} + y\hat{\jmath} = (4t^2)\hat{\imath} + (10t + t^2)\hat{\jmath}$$

Thus

$$x = 4t^2$$

When x = 16

16 = 4t^2 , t = 2 sec

Now y = 10t+t^2 at t = 2 sec

Y = 20+4 = 240 m

b)

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{v} = 10\hat{\jmath} + (8\hat{\imath} + 2\hat{\jmath})t$$

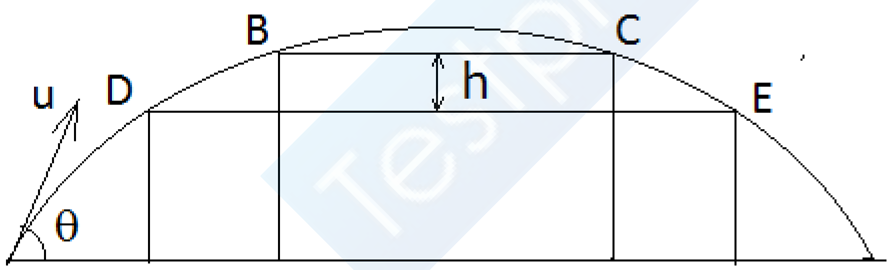
t =2 sec

$$\vec{v} = 10\hat{\jmath} + (8\hat{\imath} + 2\hat{\jmath}) \times 2$$

$$\vec{v} = 16\hat{\imath} + 14\hat{\jmath}$$

$$\overrightarrow{|v|} = \sqrt{16^2 + 14^2} = 21.26\ sec$$

Q19) A particle is projected with certain velocity as shown in figure.



If the time interval required to cross the point B and C is t_1

And the time interval required to cross D and E t_2

Find $t_2^2 - t_1^2$

Solution : We know that any projectile passes through same height for two time. Velocity of projectile along Y axis vertical is usinθ

Let point B and C be at height h_1. Thus time difference can be calculated as follows by forming equation of motion

$$h_1 = usin\theta t - \frac{1}{2}gt_1^2$$

$$gt_1^2 - 2usin\theta t + 2h_1 = 0$$

Above equation is quadratic equation having two real roots which gives time require to cross same height while going up which will have smaller value than time required to pass same height while going down

In this equation a = g , b = 2usinθ and c = 2h$_1$

Discriminate $D = \sqrt{b^2 - 4ac}$

On substituting values of a,b c in above equation

$$\sqrt{D} = \sqrt{4u^2 sin\theta^2 - 8gh_1}$$

If α and β are the two roots then difference in two roots will give us the time difference between time taken by projectile to cross the same height

$$\alpha - \beta = \frac{\sqrt{D}}{a} = \frac{\sqrt{4u^2 sin\theta^2 - 8gh_1}}{g} = \frac{2\sqrt{u^2 sin\theta^2 - 2gh_1}}{g}$$

Thus

$$t_1 = \frac{2\sqrt{u^2 sin\theta^2 - 2gh_1}}{g}$$

Similarly

$$t_2 = \frac{2\sqrt{u^2 sin\theta^2 - 2gh_2}}{g}$$

$$t_2^2 - t_1^2 = \frac{4(u^2 sin\theta^2 - 2gh_2)}{g^2} - \frac{4(u^2 sin\theta^2 - 2gh_1)}{g^2}$$

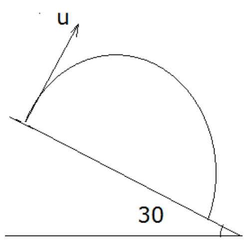
$$t_2^2 - t_1^2 = \frac{4}{g^2}(2gh_1 - 2gh_2)$$

$$t_2^2 - t_1^2 = \frac{8}{g}(h_1 - h_2)$$

Now h_1-h_2 = h as shown in figure

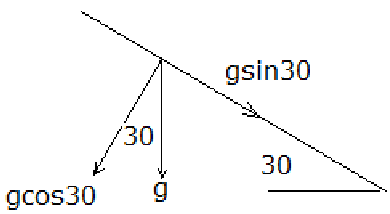
$$t_2^2 - t_1^2 = \frac{8h}{g}$$

Q20) A ball is projected from a point A with velocity 10m/s. Perpendicular to the inclined plane as shown in figure. Find the range of the ball on inclined plane



Solution:

We will turn the inclined plane and make it horizontal, and solve the problem in terms of regular projectile problem. Now gravitational acceleration will be, From figure component of gravitational acceleration perpendicular to x-axis is gcos30 and acceleration parallel to x-axis will be gsinθ



When we turn the plane then component of velocity along the plane will be zero

Now when projectile reaches the ground displacement along vertical is zero

$$0 = 10t - \frac{1}{2}gcos30t^2$$

$$t = \frac{40}{g\sqrt{3}} = \frac{40}{10\sqrt{3}} = \frac{4}{\sqrt{3}}$$

Thus time of flight of projectile is 4/√3

Now gravitation have component along slope is gsin30 as show in figure

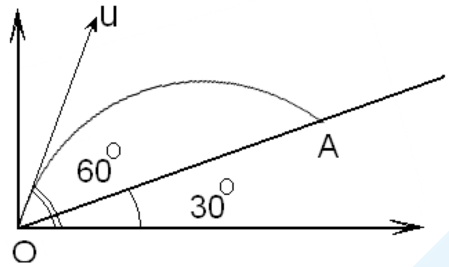
Now displacement along the slope = 0 +1/2gsinθt^2

$$R = ut + \frac{1}{2}gsin30t^2$$

$$R = 0 + \frac{1}{2}gsin30\frac{16}{3}$$

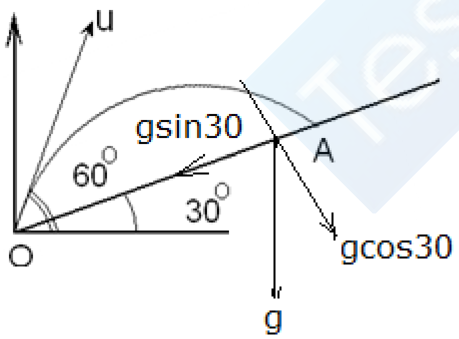
If g=10 then R= 40/3 m

Q21) A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is α =30° and the angle of barrel to the horizontal is β = 60°. The initial velocity of shell is 21 m/s. Find the distance from gun to the point at which the shell falls



Solution:

Asked to find OA



Note that projectile fired with angle of 30° with slope. Now if we rotate the plane then

Velocity along x-axis is ucos30 and velocity along y axis is usin30

Component of gravitational acceleration perpendicular to x axis is gcos30 and parallel will be gsin30

This time of flight. Displacement along y-axis is zero

$$0 = usin30t - \frac{1}{2}gcos30t^2$$

$$t = \frac{2u}{g\sqrt{3}}$$

Displacement along x axis Range

$$R = ucos30t - \frac{1}{2}gsin30t^2$$

$$R = u\frac{\sqrt{3}}{2}\frac{2u}{g\sqrt{3}} - \frac{1}{2}g\frac{1}{2}\frac{4u^2}{3g^2}$$

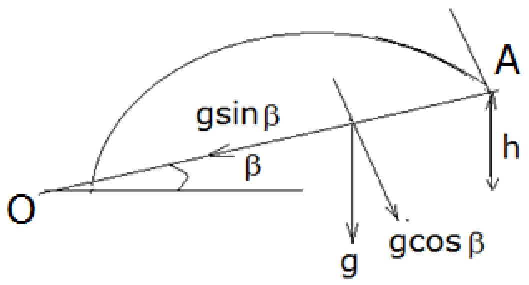
$$R = \frac{u^2}{g} - \frac{u^2}{3g}$$

$$R = \frac{u^2}{g}\left(1 - \frac{1}{3}\right) = \frac{2u^2}{3g}$$

$$R = \frac{2(21^2)}{3 \times 9.8} = 30\ m$$

Q22) A particle is projected with velocity u strikes at right angle a plane inclined at angle β with horizontal . find the height of the strike point height

Solution



As show in figure let object strike at point height h, initial velocity be u and angle of projection be θ with horizontal

Now gravitational acceleration perpendicular to inclined to plane.

If we rotate the inclined plane make it horizontal then angle of projection will be (θ – β)

Component of velocity along x aixs will be ucos(θ – β) and along y axis is usin (θ – β)

Gravitational acceleration perpendicular to plane is gcosβ and parallel to plane is gsinβ

Particle strike at 90^{O} with horizontal . i.e velocity along x axis is zero, and displacement along vertical is zero. Thus we will form two equations

At the end of time of flight t velocity along x-axis is zero form equation of motion

V = u – at

$$0 = ucos(\theta - \beta) - gsin\beta t$$

$$t = \frac{ucos(\theta - \beta)}{gsin\beta} - - - eq(1)$$

Displacement along x-axis That is OA or range

$$R = ucos(\theta - \beta)t - \frac{1}{2}gsin\beta t^2$$

Substituting value of t from equation 1 we get

$$R = ucos(\theta - \beta)\frac{ucos(\theta - \beta)}{gsin\beta} - \frac{1}{2}gsin\beta\ \frac{u^2cos^2(\theta - \beta)}{g^2sin^2\beta}$$

$$R = \frac{u^2cos^2(\theta - \beta)}{gsin\beta} - \frac{u^2cos^2(\theta - \beta)}{2gsin\beta}$$

$$R = \frac{1}{2}\frac{u^2cos^2(\theta - \beta)}{gsin\beta} - - - eq(2)$$

Since θ is assumed must be eliminated by following equation

Displacement along Y axis zero

$$0 = usin(\theta - \beta)t - \frac{1}{2}gcos\beta t^2$$

$$t = \frac{2usin(\theta - \beta)}{gsin\beta} - - - eq(3)$$

From equation 1 and 3 we get

$$\frac{2usin(\theta - \beta)}{gsin\beta} = \frac{ucos(\theta - \beta)}{gsin\beta}$$

$$tan(\theta - \beta) =$$

$$0 = ucos(\theta - \beta) - 2usin(\theta - \beta)$$

$$tan(\theta - \beta) = \frac{1}{2}cot\beta - - - eq(4)$$

From trigonometric identity

$$sec^2(\theta - \beta) = 1 + tan^2(\theta - \beta)$$

From equation 4

$$sec^2(\theta - \beta) = 1 + \frac{1}{4}cot^2\beta$$

As $sec\theta = 1/cos\theta$ thus

$$cos^2(\theta - \beta) = \frac{4}{4 + cot^2\beta} - - - eq(5)$$

Substituting equation 5 in equation 3 for range we get

$$R = \frac{1}{2}\frac{u^2 4}{gsin\beta(4 + cot^2\beta)}$$

$$R = \frac{2u^2}{gsin\beta(4 + cot^2\beta)}$$

Now from figure h = OAsinβ

$$h = \frac{2u^2}{gsin\beta(4 + cot^2\beta)}sin\beta = \frac{2u^2}{g(4 + cot^2\beta)}$$

Cot may be converted in terms of sine and cos using identity

Q23) To a man walking on straight path at speed of 3km/hr rain appears to fall vertically now man increased its speed to 6km/hr, it appears that rain is falling at 45° with vertical. Find the speed of rain Ans 3√2

Solution

It is not given the angle of rain let it be θ with vertical.

V_{rm} = Velocity of rain with respect to man

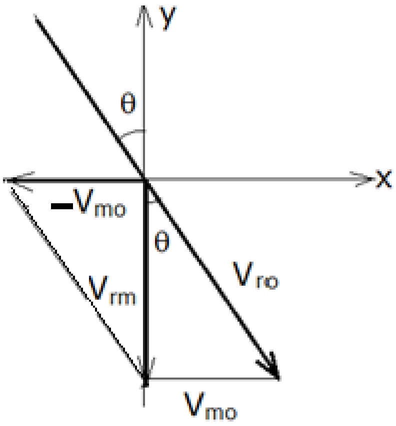
V_{ro} = Velocity of man with respect to observer

V_{mo}= Velocity of man with respect to observer

Now

$V_{rm} = V_{ro} + V_{om}$ or $V_{rm} = V_{ro} - V_{mo}$. Now refer to vector diagram

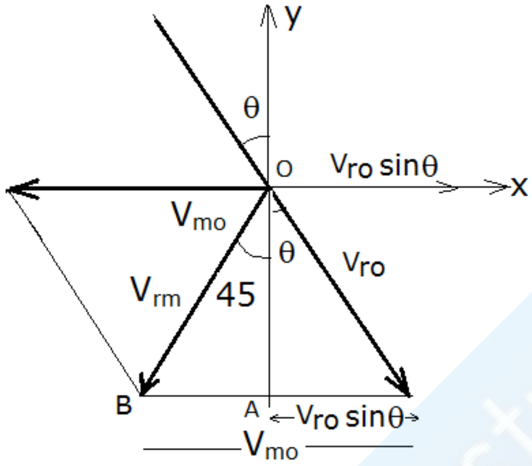
Case I When speed of man V_{mo} =3 km/hr



From figure $\sin\theta = V_{mo}/V_{rm} = 3/V_{rm}$

Or $V_{rm}\text{Si}\theta = 3$ eq(1)

CaseII When speed of man V_{mo} = 6km/hr



From adjacent figure Resolution of V_{ro} along X-axis and Y axis are $V_{ro}\sin\theta$ and $V_{ro}\text{Cos}\theta$

Also note that BA = V_{mo} - $V_{ro}\text{Sin}\theta$

BA = 6 - $V_{ro}\sin\theta$ And OA = $V_{ro}\cos\theta$

$$tan45 = \frac{BA}{OA} = \frac{6 - V_{ro}sin\theta}{V_{ro}cos\theta}$$

As tan45 = 1 ∴ $V_{ro}\cos\theta = 6 - V_{ro}\sin\theta$

from caseI equation we know that $V_{ro}\sin\theta = 3$

$V_{ro}\cos\theta = 6\text{-}3 = 3$ eq(2). By squaring and adding equation 1 and 2 we get $V_{ro} = 3\sqrt{2}$

Q24) Two cars C and D moving in the same direction on a straight road with velocity 12m/s and 10m/s respectively. When the separation in then was 200m. D starts accelerating so as to avoid accident.

Solution

Relative velocity of C with respect to D = $V_c - V_D$ = 12-10 = 2m/s or u =2m/s

Initial separation or distance is 200m

To avoid accident velocity of D should increased to 12m/s, so that relative velocity of C with respect to D become zero or v = 0

Relative acceleration of C with respect to D $a_r = 0 - a = -a$

Using formula

$v^2 = u^2 + 2a_r s$ we get

0 = 4 - 2a(200)

a_r = 0.1 m/s

Q25) A ball rolls off the top of a staircase with a horizontal velocity u m/s. if the steps are h m high and w m wide the ball will hit the edge of which step.

Solution:

By forming two individual equation in one dimension and by connecting them through time we will get necessary equation for n

Let ball hit the n^{th} step, there no initial component of velocity along vertical direction, as ball rolls in horizontal direction

Horizontal displacement =nw eq(1)

Horizontal displacement=ut eq(2)

from eq(1) and eq(2) we get ut=nw

t=nw/u eq(3)

Vertical displacement=nh eq(4)

vertical displacement=½ (g) t^2

nh=½ (g) t^2 eq(5)

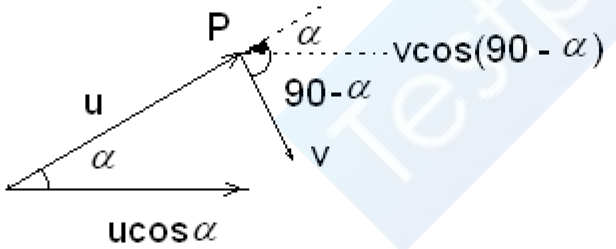
substituting value of time t from eq(3) in eq(5) we get

nh=½ (g) $(nw/u)^2$

$$n = \frac{2hu^2}{gw^2}$$

Q26) A particle is projected from a point O with velocity u in a direction making an angle α upward with the horizontal. At P, it is moving at right angle to its initial direction of projection, What will be velocity at P.

Solution



As shown in figure component of u along x-axis is ucosα

At point P it changes its direction by 90° let velocity be v

Then component of v along x-axis is vcos(90-α) = vsinα

Thus ucosα = vsinα

v = ucotα