Half-life and Average life period.

(1) **Half-life period ($T_{1/2}$ or $t_{1/2}$): Rutherford** in 1904 introduced a constant known as half-life period of the radioelement for evaluating its radioactivity or for comparing its radioactivity with the activities of other radioelements. The half-life period of a radioelement is defined, as the time required by a given amount of the element to decay to one-half of its initial value.

Mathematically, $\boxed{t_{1/2} = \frac{0.693}{\lambda}}$

Now since λ is a constant, we can conclude that half-life period of a particular radioelement is independent of the amount of the radioelement. In other words, whatever might be the amount of the radioactive element present at a time, it will always decompose to its half at the end of one half-life period.

Half-life period is a measure of the radioactivity of the element since shorter the half-life period of an element, greater is the number of the disintegrating atoms and hence greater is its radioactivity. The half-life periods or the half-lives of different radioelements vary widely, ranging from a fraction of a second to millions of years.

Fraction and Percent of radioactive nuclides left after n-Half-Lives

No. of half-lives passed (n)	Fraction of mass		Percent of mass	
	Decayed	**Left**	**Decayed**	**Left**
0	0	1.0	0	100
$\frac{1}{2}$	$\frac{\sqrt{2}-1}{\sqrt{2}} = 0.293$	$\frac{1}{\sqrt{2}} = 0.707$	29.3	79.7
1	$\frac{1}{2} = 0.50$	$\frac{1}{2} = 0.50$	50	50
2	$\frac{3}{4} = 0.75$	$\frac{1}{4} = 0.25$	75	25
3	$\frac{7}{8} = 0.875$	$\frac{1}{8} = 0.125$	87.5	12.5
4	$\frac{15}{16} = 0.9375$	$\frac{1}{16} = 0.0625$	93.75	6.25
5	$\frac{31}{32} = 0.96875$	$\frac{1}{32} = 0.03125$	96.75	3.125

∞	Total	0	100	0

Let the initial amount of a radioactive substance be N_0

After one half-life period $(t_{1/2})$ it becomes $= N_0 / 2$

After two half-life periods $(2t_{1/2})$ it becomes $= N_0 / 4$

After three half-life periods $(3t_{1/2})$ it becomes $= N_0 / 8$

After n half-life periods $(nt_{1/2})$ it shall becomes $= \left(\frac{1}{2}\right)^n N_0$

Half-life periods of some isotopes

Radio isotope	Half life		**Radio isotope**	**Half life**
$^{238}_{92}U$	4.5×10^9 years		$^{32}_{15}P$	14.3 days
$^{230}_{90}Th$	8.3×10^4 years		$^{131}_{53}I$	8.0 days
$^{226}_{88}Ra$	1.58×10^3 years		$^{214}_{84}Po$	1.5×10^{-4} seconds
$^{234}_{90}Th$	24 days		$^{14}_{6}C$	5×10^3 years
$^{59}_{26}Fe$	44.3 days		$^{222}_{86}Rn$	3.82 days

Thus, for the total disintegration of a radioactive substance an infinite time will be required.

Time (T)		**Amount of radioactive substance (N)**		**Amount of radioactive substance decomposed (N_0 – N)**
0		(N_0)		0
$t_{1/2}$		$\frac{1}{2}N_0 = \left(\frac{1}{2}\right)^1 N_0$		$\frac{1}{2}N_0 = \left[1 - \frac{1}{2}\right]N_0$
$2t_{1/2}$		$\frac{1}{4}N_0 = \left(\frac{1}{2}\right)^2 N_0$		$\frac{3}{4}N_0 = \left[1 - \frac{1}{4}\right]N_0$
$3t_{1/2}$		$\frac{1}{8}N_0 = \left(\frac{1}{2}\right)^3 N_0$		$\frac{7}{8}N_0 = \left[1 - \frac{1}{8}\right]N_0$
$4t_{1/2}$		$\frac{1}{16}N_0 = \left(\frac{1}{2}\right)^4 N_0$		$\frac{15}{16}N_0 = \left[1 - \frac{1}{16}\right]N_0$

$nt/2$		$\left(\frac{1}{2}\right)^n N_0$		$\left[1-\left(\frac{1}{2}\right)^n\right] N_0$

Amount of radioactive substance left after n half-life periods

$$\boxed{N = \left(\frac{1}{2}\right)^n N_0}$$

and Total time T $= n \times t_{1/2}$

Where n is a whole number.

(2) **Average-life period (T):**Since total decay period of any element is infinity, it is meaningless to use the term total decay period (total life period) for radioelements. Thus the term **average life** is used which the following relation determines.

Average life (T) $= \dfrac{\text{Sum of lives of the nuclei}}{\text{Total number of nuclei}}$

Relation between average life and half-life: Average life (T) of an element is the inverse of its decay constant,i.e., $T = \dfrac{1}{\lambda}$, Substituting the value of λ in the above equation,

$$\boxed{T = \frac{t_{1/2}}{0.693} = 1.44\ t_{1/2}}$$

Thus, **Average life (T)** $= 1.44 \times \text{Half life}(T_{1/2}) = \sqrt{2} \times t_{1/2}$

Thus, the average life period of a radioisotope is approximately under-root two times of its half-life period.

Note: This is because greater the value of λ, i.e., faster is the disintegration, the smaller is the average life (T).