Integrals using Euler's substitution

Integrals of the form $\int f(x), \sqrt{(ax^2 + bx + c)}\, dx$ are calculated with the aid of one of the three Euler substitution:

(1) $\sqrt{ax^2 + bx + c} = t \pm x\sqrt{a}$, if $a > 0$.

(2) $\sqrt{ax^2 + bx + c} = t\, x \pm \sqrt{c}$, if $c > 0$.

(3) $\sqrt{ax^2 + bc + c} = (x - \alpha)t$, if $ax^2 + bx + c = a(x - \alpha)(x - \beta)$, *i.e.,* if x is real root of $(ax^2 + bx + c)$.

Note: The Euler substitution often lead to rather some calculations, therefore they should be applied only when it is difficult to find another method for calculating the given integral.