

Some Integrals which cannot be found

Any function continuous on interval (a, b) has an antiderivative in that interval. In other words, there exists a function $F(x)$ such that $F'(x) = f(x)$.

However not every antiderivative $F(x)$, even when it exists is expressible in closed form in terms of elementary functions such as polynomials, trigonometric, logarithmic, exponential etc. function. Then we say that such antiderivatives or integrals "cannot be found." Some typical examples are:

$$(i) \int \frac{dx}{\log x}$$

$$(ii) \int e^{x^2} dx$$

$$(iii) \int \frac{x^2}{1+x^5} dx$$

$$(iv) \int \sqrt[3]{1+x^2} dx$$

$$(v) \int \sqrt{1+x^3} dx$$

$$(vi) \int \sqrt{1-k^2 \sin^2 x} dx$$

$$(vii) \int e^{-x^2} dx$$

$$(viii) \int \frac{\sin x}{x} dx$$

$$(ix) \int \frac{\cos x}{x} dx$$

$$(x) \int \sqrt{\sin x} dx$$

$$(xi) \int \sin(x^2) dx$$

$$(xii) \int \cos(x^2) dx$$

$$(xiii) \int x \tan x dx \text{ etc.}$$