Fundamental Integration Formulae.

(1)

(i) $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1 \qquad \because \frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$

(ii) $\int dx = x + c$

(iii) $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c,$

(iv) $\int (ax+b)^n dx = \frac{1}{a} . \frac{(ax+b)^{n+1}}{n+1} + c$

(2)

(i) $\int \frac{1}{x} dx = \log|x| + c \qquad \because \frac{d}{dx}(\log|x|) = \frac{1}{x}$

(ii) $\int \frac{1}{ax+b} dx = \frac{1}{a}(\log|ax+b| + c$

(3) $\int e^x dx = e^x + c \qquad \because \frac{d}{dx}(e^x) = e^x$

(4) $\int a^x dx = \frac{a^x}{\log_e a} + c \qquad \because \frac{d}{dx}\left(\frac{a^x}{\log_e a}\right) = a^x$

(5) $\int \sin x \, dx = -\cos x + c \qquad \because \frac{d}{dx}(-\cos x) = \sin x$

(6) $\int \cos x \, dx = \sin x + c \qquad \because \frac{d}{dx}(\sin x) = \cos x$

(7) $\int \sec^2 x \, dx = \tan x + c \qquad \because \frac{d}{dx}(\tan x) = \sec^2 x$

(8) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c \qquad \because \frac{d}{dx}(-\cot x) = \operatorname{cosec}^2 x$

(9) $\int \sec x \tan x \, dx = \sec x + c$ $\quad \because \frac{d}{dx}(\sec x) = \sec x \tan x$

(10) $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$ $\quad \because \frac{d}{dx}(-\operatorname{cosec} x) = \operatorname{cosec} x \cot x$

(11) $\int \tan x \, dx = -\log|\cos x| + c = \log|\sec x| + c$ $\quad \because \frac{d}{dx}(\log \cos x) = -\tan x$

(12) $\int \cot x \, dx = \log|\sin x| + c = -\log|\operatorname{cosec} x| + c$ $\quad \because \frac{d}{dx}(\log \sin x) = \cot x$

(13) $\int \sec x \, dx = \log|\sec x + \tan x| + c = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$ $\because \frac{d}{dx}\log(\sec x + \tan x) = \sec x$

(14) $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + c = \log \tan \frac{x}{2} + c$ $\because \frac{d}{dx}(\log|\operatorname{cosec} x - \cot x|) = \operatorname{cosec} x$

(15) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c = -\cos^{-1} x + c$ $\quad \because \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

(16) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c = -\cos^{-1} \frac{x}{a} + c$ $\quad \because \frac{d}{dx}\left(\sin^{-1} \frac{x}{a}\right) = \frac{1}{\sqrt{a^2-x^2}}$,
$\frac{d}{dx}(\cos^{-1} \frac{x}{a}) = \frac{-1}{\sqrt{a^2-x^2}}$

(17) $\int \frac{dx}{1+x^2} = \tan^{-1} x + c = -\cot^{-1} x + c$ $\quad \because \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$,
$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

(18) $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c = \frac{-1}{a} \cot^{-1} \frac{x}{a} + c$ $\quad \because \frac{d}{dx}\left(\tan^{-1} \frac{x}{a}\right) = \frac{a}{a^2+x^2}$
$\frac{d}{dx}\left(\cot^{-1} \frac{x}{a}\right) = \frac{-a}{a^2+x^2}$

(19) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x + c = -\operatorname{cosec}^{-1}x + c \because \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}\ \frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{x\sqrt{x^2-1}}$

(20) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c = \frac{-1}{a}\operatorname{cosec}^{-1}\frac{x}{a} + c \quad \because \frac{d}{dx}\left(\sec^{-1}\frac{x}{a}\right) = \frac{a}{x\sqrt{x^2-a^2}}$

$\frac{d}{dx}(\operatorname{cosec}^{-1}\frac{x}{a}) = \frac{-a}{x\sqrt{x^2-a^2}}$

Note: In any of the fundamental integration formulae, if x is replaced by $ax + b$, then the same formulae is applicable but we must divide by coefficient of x or derivative of ($ax + b$) *i.e., a*. In general, if

$\int f(x)dx = \phi(x) + c$, then $\int f(ax+b)dx = \frac{1}{a}\phi(ax+b) + c$

$\int \sin(ax+b)dx = \frac{-1}{a}\cos(ax+b) + c, \quad \int \sec(ax+b)dx = \frac{1}{a}\log|\sec(ax+b) + \tan(ax+b)| + c$ *etc.*

Some more results:

(i) $\int \frac{1}{x^2-a^2} = \frac{-1}{a}\coth^{-1}\frac{x}{a} + c = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$, when $x > a$

(ii) $\int \frac{1}{a^2-x^2}dx = \frac{1}{a}\tanh^{-1}\frac{x}{a} + c = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$, when $x < a$

(iii) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\{|x + \sqrt{x^2-a^2}|\} + c = \cosh^{-1}\left(\frac{x}{a}\right) + c$

(iv) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\{|x + \sqrt{x^2+a^2}|\} + c = \sinh^{-1}\left(\frac{x}{a}\right) + c$

(v) $\int \sqrt{a^2-x^2}dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\sin^{-1}\left(\frac{x}{a}\right) + c$

(vi) $\int \sqrt{x^2-a^2}dx = \frac{1}{2}x\sqrt{x^2-a^2} - \frac{1}{2}a^2\log\{x + \sqrt{x^2-a^2}\} + c = \frac{1}{2}x\sqrt{x^2-a^2} - \frac{1}{2}a^2\cosh^{-1}\left(\frac{x}{a}\right) + c$

(vii) $\int \sqrt{x^2 + a^2}\, dx = \frac{1}{2} x \sqrt{x^2 + a^2} + \frac{1}{2} a^2 \log\{x + \sqrt{x^2 + a^2}\} + c = \frac{1}{2} x \sqrt{x^2 + a^2} + \frac{1}{2} a^2 \sinh^{-1}\left(\frac{x}{a}\right) + c$

Important Tips

- *The signum function has an antiderivative on any interval which doesn't contain the point $x = 0$, and does not possess an anti-derivative on any interval which contains the point.*
- *The antiderivative of every odd function is an even function and vice-versa*