Integration by Substitution.

(1) **When integrand is a function *i.e.,*** $\int f[\phi(x)]\phi'(x)\,dx$ **:**

Here, we put $\phi(x)=t,$ so that $\phi'(x)dx = dt$ and in that case the integrand is reduced to $\int f(t)dt$.

In this method, the integrand is broken into two factors so that one factor can be expressed in terms of the function whose differential coefficient is the second factor.

(2) **When integrand is the product of two factors such that one is the derivative of the others *i.e.,*** $I=\int f'(x).\,f(x).\,dx$

In this case we put $f(x)=t$ and convert it into a standard integral.

(3) **Integral of a function of the form $f(ax+b)$:** Here we put $ax+b=t$ and convert it into standard integral. Obviously if $\int f(x)dx = \phi(x),$ then, $\int f(ax+b)dx = \frac{1}{a}\phi(ax+b)+c$

(4) **If integral of a function of the form** $\frac{f'(x)}{f(x)}dx = \log[\,f(x)]+c$

(5) **If integral of a function of the form,** $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1}+c\ [n\neq -1]$

(6) **If the integral of a function of the form,** $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)}+c$

(7) **Standard substitutions**

	Integrand form	**Substitution**

(i)	$\sqrt{a^2-x^2}$, $\dfrac{1}{\sqrt{a^2-x^2}}$, a^2-x^2	$x=a\sin\theta$, $x=a\cos\theta$
(ii)	$\sqrt{x^2+a^2}$, $\dfrac{1}{\sqrt{x^2+a^2}}$, x^2+a^2	$x=a\tan\theta$ or $x=a\sinh\theta$
(iii)	$\sqrt{x^2-a^2}$, $\dfrac{1}{\sqrt{x^2-a^2}}$, x^2-a^2	$x=a\sec\theta$ or $x=a\cosh\theta$
(iv)	$\sqrt{\dfrac{x}{a+x}}$, $\sqrt{\dfrac{a+x}{x}}$, $\sqrt{x(a+x)}$, $\dfrac{1}{\sqrt{x(a+x)}}$	$x=a\tan^2\theta$
(v)	$\sqrt{\dfrac{x}{a-x}}$, $\sqrt{\dfrac{a-x}{x}}$, $\sqrt{x(a-x)}$, $\dfrac{1}{\sqrt{x(a-x)}}$	$x=a\sin^2\theta$
(vi)	$\sqrt{\dfrac{x}{x-a}}$, $\sqrt{\dfrac{x-a}{x}}$, $\sqrt{x(x-a)}$, $\dfrac{1}{\sqrt{x(x-a)}}$	$x=a\sec^2\theta$
(vii)	$\sqrt{\dfrac{a-x}{a+x}}$, $\sqrt{\dfrac{a+x}{a-x}}$	$x=a\cos 2\theta$
(viii)	$\sqrt{\dfrac{x-\alpha}{\beta-x}}$, $\sqrt{(x-\alpha)(\beta-x)}$, $(\beta>\alpha)$	$x=\alpha\cos^2\theta+\beta\sin^2\theta$