Procedure of Curve Sketching.

(1) **Symmetry:**

(i) Symmetry about *x*-axis: If all powers of y in equation of the given curve are even, then it is symmetric about *x*-axis *i.e.,* the shape of the curve above *x*-axis is exactly identical to its shape below *x*-axis.

For example, $y^2 = 4ax$ is symmetric about *x*-axis.

(ii) Symmetry about y-axis: If all power of x in the equation of the given curve are even, then it is symmetric about *y*-axis

For example, $x^2 = 4ay$ is symmetric about *y*-axis.

(iii) Symmetry in opposite quadrants or symmetry about origin: If by putting $-x$ for x and $-y$ for y, the equation of a curve remains same, then it is symmetric in opposite quadrants.

For example, $x^2 + y^2 = a^2$ and $xy = a^2$ are symmetric in opposite quadrants.

(iv) Symmetry about the line $y = x$: If the equation of a given curve remains unaltered by interchanging x and y then it is symmetric about the line $y = x$ which passes through the origin and makes an angle of 45^0 with the positive direction of *x*-axis.

(2) **Origin:** If the equation of curve contains no constant terms then it passes through the origin. Find whether the curve passes through the origin or not.

For examples, $x^2 + y^2 + 4ax = 0$ passes through origin.

(3) **Points of intersection with the axes:** If we get real values of x on putting $y = 0$ in the equation of the curve, then real values of x and $y = 0$ give those points where the curve

cuts the *x*-axis. Similarly by putting $x = 0,$ we can get the points of intersection of the curve and *y*-axis.

For example, the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersect the axes at points $(\pm a, 0)$ and $(0, \pm b)$.

(4) **Special points:** Find the points at which $\frac{dy}{dx} = 0$, at these points the tangent to the curve is parallel to *x*-axis. Find the points at which $\frac{dx}{dy} = 0$. At these points the tangent to the curve is parallel to *y*-axis.

(5) **Region:** Write the given equation as $y = f(x)$, and find minimum and maximum values of *x* which determine the region of the curve.

For example for the curve $xy^2 = a^2(a - x) \Rightarrow y = a\sqrt{\frac{a - x}{x}}$

Now *y* is real, if $0 \le x \le a$, So its region lies between the lines *x* = 0 and *x* = *a*

(6) **Regions where the curve does not exist:** Determine the regions in which the curve does not exists. For this, find the value of *y* in terms of *x* from the equation of the curve and find the value of *x* for which *y* is imaginary. Similarly find the value of *x* in terms of *y* and determine the values of *y* for which *x* is imaginary. The curve does not exist for these values of *x* and *y.*

For example, the values of *y* obtained from $y^2 = 4ax$ are imaginary for negative value of *x*, so the curve does not exist on the left side of *y*-axis. Similarly the curve $a^2y^2 = x^2(a - x)$ does not exist for $x > a$ as the values of *y* are imaginary for $x > a$.