Formation of Differential Equation.

Formulating a differential equation from a given equation representing a family of curves means finding a differential equation whose solution is the given equation. If an equation, representing a family of curves, contains *n* arbitrary constants, then we differentiate the given equation *n* times to obtain *n* more equations. Using all these equations, we eliminate the constants. The equation so obtained is the differential equation of order *n* for the family of given curves.
Consider a family of curves $f(x, y, a_1, a_2 \ldots.., a_n) = 0$ ......(i)
Where $a_1, a_2, \ldots\ldots a_n$ are *n* independent parameters.

Equation (i) is known as an *n* parameter family of curves *e.g. y = mx* is a one-parameter family of straight lines. $x^2 + y^2 + ax + by = 0$ is a two parameters family of circles.

If we differentiate equation (i) *n* times *w.r.t. x*, we will get *n* more relations between $x, y, a_1, a_2, \ldots.. a_n$ and derivatives of *y w.r.t. x*. By eliminating $a_1, a_2, \ldots., a_n$ from these *n* relations and equation (i), we get a differential equation.
Clearly order of this differential equation will be *n i.e.* equal to the number of independent parameters in the family of curves.

Algorithm for formation of differential equations

Step (i)**:**Write the given equation involving independent variable *x* (say), dependent variable *y* (say) and the arbitrary constants.

Step (ii)**:**Obtain the number of arbitrary constants in step (i). Let there be n arbitrary constants.

Step (iii)**:**Differentiate the relation in step (i) *n* times with respect to *x*.

Step (iv):Eliminate arbitrary constants with the help of n equations involving differential coefficients obtained in step (iii) and an equation in step (i). The equation so obtained is the desired differential equation.