Variable Separable type Differential Equation.

(1) **Solution of differential equations:**If we have a differential equation of order '*n*' then by solving a differential equation we mean to get a family of curves with *n* parameters whose differential equation is the given differential equation. Solution or integral of a differential equation is a relation between the variables, not involving the differential coefficients such that this relation and the derivatives obtained from it satisfy the given differential equation. The solution of a differential equation is also called its primitive.

For example $y = e^x$ is a solution of the differential equation $\frac{dy}{dx} = y$.

(i) **General solution:**The solution which contains as many as arbitrary constants as the order of the differential equation is called the general solution of the differential equation. For example, $y = A\cos x + B\sin x$ is the general solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$. But $y = A\cos x$ is not the general solution as it contains one arbitrary constant.

(ii) **Particular solution**: Solution obtained by giving particular values to the arbitrary constants in the general solution of a differential equation is called a particular solution. For example, $y = 3\cos x + 2\sin x$ is a particular solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$

(2) **Differential equations of first order and first degree:**A differential equation of first order and first degree involves *x*, *y*and $\frac{dy}{dx}$. So it can be put in any one of the following forms:

$\frac{dy}{dx} = f(x,y)$ or $f\left(x, y, \frac{dy}{dx}\right) = 0$ or $f(x,y)dx + g(x,y)dy = 0$ where $f(x,y)$ and $g(x,y)$ are obviously the functions of *x* and y.

(3) **Geometrical interpretation of the differential equations of first order and first degree**:

The general form of a first order and first degree differential equation is $f\left(x, y, \frac{dy}{dx}\right) = 0$ .....(i)

We know that the direction of the tangent of a curve in Cartesian rectangular coordinates at any point is given by $\frac{dy}{dx}$, so the equation in (i) can be known as an equation which establishes the relationship between the coordinates of a point and the slope of the tangent *i.e.*, $\frac{dy}{dx}$ to the integral curve at that point. Solving the differential equation given by (i) means finding those curves for which the direction of

tangent at each point coincides with the direction of the field. All the curves represented by the general solution when taken together will give the locus of the differential equation. Since there is one arbitrary constant in the general solution of the equation of first order, the locus of the equation can be said to be made up of single infinity of curves.

(4) **Solution of first order and first degree differential equations:** A first order and first degree differential equation can be written as

$f(x, y)dx + g(x, y)dy = 0$

or $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)'}$ or $\frac{dy}{dx} = \phi(x,y)$

Where $f(x,y)$ and $g(x,y)$ are obviously the functions of x and y. It is not always possible to solve this type of equations. The solution of this type of differential equations is possible only when it falls under the category of some standard forms.

(5) **Equations in variable separable form :** If the differential equation of the form

$f_1(x)dx = f_2(y)dy$ .....(i)

Where f_1 and f_2 being functions of x and y only. Then we say that the variables are separable in the differential equation.

Thus, integrating both sides of (i), we get its solution as $\int f_1(x)dx = \int f_2(y)dy + C$,

Where c is an arbitrary constant.

There is no need of introducing arbitrary constants to both sides as they can be combined together to give just one.

(i) **Differential equations of the type** $\frac{dy}{dx} = f(x)$

To solve this type of differential equations we integrate both sides to obtain the general solution as discussed following: $\frac{dy}{dx} = f(x) \Leftrightarrow dy = f(x)dx$

Integrating both sides, we obtain, $\int dy = \int f(x)dx + C$ or $y = \int f(x)dx + C$.

(ii) **Differential equations of the type** $\frac{dy}{dx} = f(y)$

To solve this type of differential equations we integrate both sides to obtain the general solution as discussed following : $\frac{dy}{dx} = f(y) \Rightarrow \frac{dx}{dy} = \frac{1}{f(y)} \Rightarrow dx = \frac{1}{f(y)} dy$

Integrating both sides, we obtain, $\int dx = \int \frac{1}{f(y)} dy + C$ or $x = \int \frac{1}{f(y)} dy + C$.

(6) **Equations reducible to variable separable form**

(i) Differential equations of the form $\frac{dy}{dx} = f(ax + by + c)$ can be reduced to variable separable form by the substitution $ax + by + c = Z$

$\therefore a + b\frac{dy}{dx} = \frac{dZ}{dx}$

$\therefore \left(\frac{dZ}{dx} - a\right)\frac{1}{b} = f(Z) \Rightarrow \frac{dZ}{dx} = a + bf(Z)$.

This is variable separable form.

(ii) **Differential equation of the form**

$\frac{dy}{dx} = \frac{ax + by + c}{Ax + By + C}$, where $\frac{a}{A} = \frac{b}{B} = K$ (say)

$\therefore \frac{dy}{dx} = \frac{K(Ax + By) + C}{Ax + By + C}$

Put $Ax + By = Z$

$\therefore \quad A + B\frac{dy}{dx} = \frac{dZ}{dx}, \quad \therefore \left[\frac{dZ}{dx} - A\right]\frac{1}{B} = \frac{KZ + C}{Z + C} \Rightarrow \frac{dZ}{dx} = A + B\frac{KZ + C}{Z + C}$

This is variable separable form and can be solved.