

## Miscellaneous Differential Equation.

(1) **A special type of second order differential equation:**  $\frac{d^2y}{dx^2} = f(x)$  .....(i)

Equation (i) may be re-written as  $\frac{d}{dx} \left( \frac{dy}{dx} \right) = f(x) \Rightarrow d \left( \frac{dy}{dx} \right) = f(x) dx$

Integrating,  $\frac{dy}{dx} = \int f(x) dx + c_1$  *i.e.*  $\frac{dy}{dx} = F(x) + c_1$  .....(ii)

Where  $F(x) = \int f(x) dx + c_1 dx$

From (ii),  $dy = f(x) dx + c_1 dx$

Integrating,  $y = \int F(x) dx + c_1 x + c_2$

$\therefore y = H(x) + c_1 x + c_2$

Where  $\boxed{H(x) = \int F(x) dx}$   $c_1$  and  $c_2$  are arbitrary constants.

(2) **Particular solution type problems:** To solve such a problem, we proceed according to the type of the problem (*i.e.* variable-separable, linear, exact, homogeneous etc.) and then we apply the given conditions to find the particular values of the arbitrary constants.