Laws of Indices.

(1) $a^0 = 1$, $(a \neq 0)$

(2) $a^{-m} = \frac{1}{a^m}$, $(a \neq 0)$

(3) $a^{m+n} = a^m . a^n$, Where *m* and *n* are rational numbers

(4) $a^{m-n} = \frac{a^m}{a^n}$, Where *m* and *n* are rational numbers, $a \neq 0$

(5) $(a^m)^n = a^{mn}$

(6) $a^{p/q} = \sqrt[q]{a^p}$

(7) If $x = y$, then $a^x = a^y$, but the converse may not be true.

For example: $(1)^6 = (1)^8$, but $6 \neq 8$

If $a \neq \pm 1$, or 0, then $x = y$

(ii) If $a = 1$, then x, y may be any real number

(iii) If $a = -1$, then x, y may be both even or both odd

(iv) If $a = 0$, then x, y may be any non-zero real number

But if we have to solve the equations like $[f(x)]^{\phi(x)} = [f(x)]^{\Psi(x)}$ then we have to solve :

(a) $f(x) = 1$

(b) $f(x) = -1$

(c) $f(x) = 0$

(d) $\phi(x) = \Psi(x)$

Verification should be done in (b) and (c) cases

(8) $a^m . b^m = (ab)^m$ is not always true

In real domain, $\sqrt{a}\sqrt{b} = \sqrt{(ab)}$, only when $a \geq 0, b \geq 0$

In complex domain, $\sqrt{a}.\sqrt{b} = \sqrt{(ab)}$, if at least one of a and b is positive.

(9) If $a^x = b^x$ then consider the following cases:

(i) If $a \neq \pm b,$ then $x = 0$

(ii) If $a = b \neq 0,$ then x may have any real value

(iii) If $a = -b$, then x is even.

If we have to solve the equation of the form $[f(x)]^{\phi(x)} = [g(x)]^{\phi(x)}$ *i.e.,* same index, different bases, then we have to solve

(a) $f(x) = g(x)$,

(b) $f(x) = -g(x)$,

(c) $\phi(x) = 0$

Verification should be done in (b) and (c) cases.