Rationalization Factors.

If two surds be such that their product is rational, then each one of them is called rationalizing factor of the other.

Thus each of $2\sqrt{3}$ and $\sqrt{3}$ is a rationalizing factor of each other. Similarly $\sqrt{3}+\sqrt{2}$ and $\sqrt{3}-\sqrt{2}$ are rationalizing factors of each other, as $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})=1$, which is rational.

To find the factor which will rationalize any given binomial surd:

Case I: Suppose the given surd is $\sqrt[p]{a}-\sqrt[q]{b}$

Suppose $a^{1/P}=x, b^{1/q}=y$ and let *n* be the L.C.M. of *p* and *q*. Then x^n and y^n are both rational.

Now x^n-y^n is divisible by $x-y$ for all values of *n*, and

$x^n-y^n=(x-y)(x^{n-1}+x^{n-2}y+x^{n-3}y^2+.....+y^{n-1})$.

Thus the rationalizing factor is $x^{n-1}+x^{n-2}y+x^{n-3}y^2+.....+y^{n-1}$ and the rational product is x^n-y^n.

Case II: Let the given surd be $\sqrt[p]{a}+\sqrt[q]{b}$.

Let x, y, n have the same meaning as in Case I.

(1) If *n* is even, then x^n-y^n is divisible by *x+ y* and

$x^n-y^n=(x+y)(x^{n-1}-x^{n-2}y+x^{n-3}y^3-.....-y^{n-1})$

Thus the rationalizing factor is $x^{n-1}-x^{n-2}y+x^{n-3}y^2-.....-y^{n-1}$ and the rational product is x^n-y^n.

(2) If *n* is odd, x^n+y^n is divisible by $x+y$, and $x^n+y^n=(x+y)(x^{n-1}-x^{n-2}y+....+y^{n-1})$

Thus the rationalizing factor is $x^{n-1}-x^{n-2}y+.....-xy^{n-2}+y^{n-1}$ and the rational product is x^n+y^n.