

Cube Root of a Binomial Quadratic Surd.

If $(a + \sqrt{b})^{1/3} = x + \sqrt{y}$ then $(a - \sqrt{b})^{2/3} = x - \sqrt{y}$, where a is a rational number and b is a surd.

Procedure of finding $(a + \sqrt{b})^{1/3}$ is illustrated with the help of an example:

Taking $(37 - 30\sqrt{3})^{1/3} = x + \sqrt{y}$ we get on cubing both sides, $37 - 30\sqrt{3}$

$$= x^3 + 3xy - (3x^2 + y)\sqrt{y}$$

$$\therefore x^3 + 3xy = 37$$

$$(3x^2 + y)\sqrt{y} = 30\sqrt{3} = 15\sqrt{12}$$

As $\sqrt{3}$ cannot be reduced, let us assume $y = 3$ we get $3x^2 + y = 3x^2 + 3 = 30 \therefore x = 3$

Which doesn't satisfy $x^3 + 3xy = 37$

Again taking $y = 12$, we get

$$3x^2 + 12 = 15, \therefore x = 1$$

$$x = 1, y = 12 \text{ Satisfy } x^3 + 3xy = 37$$

$$\therefore \sqrt[3]{37 - 30\sqrt{3}} = 1 - \sqrt{12} = 1 - 2\sqrt{3}$$