Definition.

An expression of the form $\dfrac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial in x, is called a rational fraction.

(1) **Proper rational functions:** Functions of the form $\dfrac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomials and $g(x) \neq 0$, are called rational functions of x.

If degree of $f(x)$ is less than degree of $g(x)$, then $\dfrac{f(x)}{g(x)}$ is called a proper rational function.

Example: $\dfrac{x+2}{x^2 + 2x + 4}$ is a proper rational function.

(2) **Improper rational functions:** If degree of $f(x)$ is greater than or equal to degree of $g(x)$, then $\dfrac{f(x)}{g(x)}$ is called an improper rational function.

For example: $\dfrac{x^3}{(x-1)(x-2)}$ is an improper rational function.

(3) **Partial fractions:** Any proper rational function can be broken up into a group of different rational fractions, each having a simple factor of the denominator of the original rational function. Each such fraction is called a partial fraction.

If by some process, we can break a given rational function $\dfrac{f(x)}{g(x)}$ into different fractions, whose denominators are the factors of $g(x)$, then the process of obtaining them is called the resolution or decomposition of $\dfrac{f(x)}{g(x)}$ into its partial fractions.