Different Cases of Partial Fractions.

(1) **When the denominator consists of non-repeated linear factors:** To each linear factor $(x-a)$ occurring once in the denominator of a proper fraction, there corresponds a single partial fraction of the form $\frac{A}{x-a}$, where *A* is a constant to be determined.

If $g(x) = (x-a_1)(x-a_2)(x-a_3).......(x-a_n)$, then we assume that,

$$\frac{f(x)}{g(x)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + ... + \frac{A_n}{x-a_n}$$

Where $A_1, A_2, A_3 .... A_n$ are constants, can be determined by equating the numerator of L.H.S. to the numerator of R.H.S. (after L.C.M.) and substituting $x = a_1, a_2 ...... a_n$.

Note: Remainder of polynomial $f(x)$, when divided by $(x-a)$ is $f(a)$.

e.g., Remainder of x^2+3x-7 ,when divided by $x-2$ is $(2)^2+3(2)-7=3$.

❑ $\frac{px+q}{(x-a)(x-b)} = \frac{pa+q}{(x-a)(a-b)} + \frac{pb+q}{(b-a)(x-b)}$

(2) **When the denominator consists of linear factors, some repeated:** To each linear factor (x – a) occurring r times in the denominator of a proper rational function, there corresponds a sum of r partial fractions.

Let $g(x) = (x-a)^k (x-a_1)(x-a_2).......(x-a_r)$.Then we assume that

$$\frac{f(x)}{g(x)} = \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + ... + \frac{A_k}{(x-a)^k} + \frac{B_1}{(x-a_1)} + ... + \frac{B_r}{(x-a_r)}$$

Where $A_1, A_2, ..., A_k$ are constants. To determined the value of constants adopt the procedure as above.

(3) **When the denominator consists of non-repeated quadratic factors:** To each irreducible non repeated quadratic factor ax^2+bx+c, there corresponds a partial fraction of the form $\frac{Ax+B}{ax^2+bx+c}$, where A and B are constants to be determined.

Note: $$\frac{px+q}{x^2(x-a)} = \frac{-q}{ax^2} - \frac{pa+q}{a^2x} + \frac{pa+q}{a^2(x-a)}$$

- $$\frac{px+q}{x(x-a)^2} = \frac{q}{a^2x} - \frac{q}{a^2(x-a)} + \frac{pa+q}{a(x-a)^2}$$

- $$\frac{px+q}{x(x^2+a^2)} = \frac{q}{a^2x} + \frac{pa^2-qx}{a^2(x^2+a^2)}$$

(4) **When the denominator consists of repeated quadratic factors:** To each irreducible quadratic factor ax^2+bx+c occurring r times in the denominator of a proper rational fraction there corresponds a sum of r partial fractions of the form.

$$\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \ldots\ldots\ldots + \frac{A_rx+B_r}{(ax^2+bx+c)^r}$$

Where, A's and B's are constants to be determined.