Logarithm of a Complex Number.

Let $z = x + iy$ and

$\log_e(x+iy) = a + ib$ .....(i)

$x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$ .....(ii)

then $x = r\cos\theta,\ y = r\sin\theta$, clearly $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

From equation (ii), $\log(x+iy) = \log_e(re^{i\theta}) = \log r + \log_e e^{i\theta} = \log_e r + i\theta$

$= \log_e \sqrt{(x^2 + y^2)} + i\tan^{-1}\left(\frac{y}{x}\right)$

$\boxed{\log_e(z) = \log_e |z| + i\,amp\ z}$

Obviously, the general value is $Log(z) = \log_e(z) + 2\pi ni$ $\quad (-\pi < amp(z) < \pi)$