Triangle Inequalities.

In any triangle, sum of any two sides is greater than the third side and difference of any two side is less than the third side. By applying this basic concept to the set of complex numbers we are having the following results.

(1) $|z_1 + z_2| \le |z_1| + |z_2|$

(2) $|z_1 - z_2| \le |z_1| + |z_2|$

(3) $|z_1 + z_2| \ge ||z_1| - |z_2||$

(4) $|z_1 - z_2| \ge ||z_1| - |z_2||$

Note: In a complex plane $|z_1 - z_2|$ is the distance between the points z_1 and z_2.

The equality $|z_1 + z_2| = |z_1| + |z_2|$ holds only when $\arg(z_1) = \arg(z_2)$ i.e., z_1 and z_2 are parallel.

The equality $|z_1 - z_2| = ||z_1| - |z_2||$ holds only when $\arg(z_1) - \arg(z_2) = \pi$ i.e., z_1 and z_2 are antiparallel.

In any parallelogram sum of the squares of its sides is equal to the sum of the squares of its diagonals i.e. $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

Law of polygon i.e., $|z_1 + z_2 + ... + z_n| \le |z_1| + |z_2| + ... + |z_n|$

Important Tips

☞ The area of the triangle whose vertices are z, iz and z + izis $\frac{1}{2}|z|^2$.

☞ If z_1, z_2, z_3 be the vertices of a triangle then the area of the triangle is $\dfrac{\sum (z_2 - z_3)|z_1|^2}{4iz_1}$.

☞ Area of the triangle with vertices z, wz and $z + wz$ is $\dfrac{\sqrt{3}}{4}|z^2|$.

☞ If z_1, z_2, z_3 be the vertices of an equilateral triangle and z_o be the circumcentre, then $z_1^2 + z_2^2 + z_3^2 + = 3z_0^2$.

☞ If $z_1, z_2, z_3 .....z_n$ be the vertices of a regular polygon of n sides and z_0 be its centroid, then $z_1^2 + z_2^2 + ... + z_n^2 = nz_0^2$.

☞ If z_1, z_2, z_3 be the vertices of a triangle, then the triangle is equilateral iff $(z_1 - z_2)^2 + (z_2 - z_3)^2 + (z_3 - z_1)^2 = 0$ or $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$ or $\dfrac{1}{z_1 - z_2} + \dfrac{1}{z_2 - z_3} + \dfrac{1}{z_3 - z_1} = 0$.

☞ If $z_1, z_2 z_3$ are the vertices of an isosceles triangle, right angled at z_2 then $z_1^2 + z_2^2 + z_3^2 = 2z_2(z_1 + z_3)$.

☞ If z_1, z_2, z_3 are the vertices of right-angled isosceles triangle, then $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$.

☞ If one of the vertices of the triangle is at the origin i.e., $z_3 = 0,$ then the triangle is equilateral iff $z_1^2 + z_2^2 - z_1 z_2 = 0$.

☞ If z_1, z_2, z_3 and z_1', z_2', z_3' are the vertices of a similar triangle, then $\begin{vmatrix} z_1 & z_1' & 1 \\ z_2 & z_2' & 1 \\ z_3 & z_3' & 1 \end{vmatrix} = 0$.

☞ If z_1, z_2, z_3 be the affixes of the vertices A, B, C respectively of a triangle ABC, then its orthocentreis $\dfrac{a(\sec A)z_1 + b(\sec B)z_2 + (c \sec C)z_3}{a \sec A + b \sec B + c \sec C}$.