Standard Loci in the Argand Plane.

(1) If z is a variable point in the argand plane such that $\arg(z) = \theta$, then locus of z is a straight line (excluding origin) through the origin inclined at an angle θ with x–axis.

(2) If z is a variable point and z_1 is a fixed point in the argand plane such that $\arg(z - z_1) = \theta$, then locus of z is a straight line passing through the point representing z_1 and inclined at an angle θ with x-axis. Note that the point z_1 is excluded from the locus.

(3) If z is a variable point and z_1, z_2 are two fixed points in the argand plane, then

(i) $|z - z_1| = |z - z_2|$ $\Rightarrow$ Locus of z is the perpendicular bisector of the line segment joining z_1 and z_2

(ii) $|z - z_1| + |z - z_2|$ = constant $(\neq |z_1 - z_2|)$ $\Rightarrow$ Locus of z is an ellipse

(iii) $|z - z_1| + |z - z_2| = |z_1 - z_2|$ $\Rightarrow$ Locus of z is the line segment joining z_1 and z_2

(iv) $|z - z_1| - |z - z_2| = |z_1 - z_2|$ $\Rightarrow$ Locus of z is a straight line joining z_1 and z_2 but z does not lie between z_1 and z_2.

(v) $|z - z_1| - |z - z_2| = \text{constant} \quad (\neq |z_1 - z_2|)$ $\Rightarrow$ Locus of z is a hyperbola.

(vi) $|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|$ $\Rightarrow$ Locus of z is a circle with z_1 and z_2 as the extremities of diameter.

(vii) $|z - z_1| = k|z - z_2| \quad k \neq 1$ $\Rightarrow$ Locus of z is a circle.

(viii) $\arg\left(\frac{z - z_1}{z - z_2}\right) = \alpha \ \text{(fixed)}$ $\Rightarrow$ Locus of z is a segment of circle.

(ix) $\arg\left(\frac{z - z_1}{z - z_2}\right) = \pm\pi/2$ $\Rightarrow$ Locus of z is a circle with z_1 and z_2 as the vertices of diameter.

(x) $\arg\left(\frac{z - z_1}{z - z_2}\right) = 0$ or π $\Rightarrow$ Locus z is a straight line passing through z_1 and z_2.

(xi) The equation of the line joining complex numbers z_1 and z_2 is given by $\frac{z - z_1}{z_2 - z_1} = \frac{\bar{z} - \bar{z}_1}{\bar{z}_2 - \bar{z}_1}$ or

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$$