Introduction.

Number system consists of real numbers $(-5, 7, \frac{1}{3}, \sqrt{3} \ldots\ldots\ldots \ldots etc.)$ and imaginary numbers $(\sqrt{-5}, \sqrt{-9}$ ....etc.) If we combine these two numbers by some mathematical operations, the resulting number is known as Complex Number *i.e.*, "Complex Number is the combination of real and imaginary numbers".

(1) **Basic concepts of complex number**

(i) **General definition:** A number of the form $x + iy$, where $x, y \in R$ and $i = \sqrt{-1}$ is called a complex number so the quantity $\sqrt{-1}$ is denoted by '*i*' called iota thus $i = \sqrt{-1}$.

A complex number is usually denoted by *z* and the set of complex number is denoted by *c*

i.e., $c = \{x + iy : x \in R, y \in R, i = \sqrt{-1}\}$

For example, $5 + 3i, -1 + i, 0 + 4i, 4 + 0i$ etc. are complex numbers.

Note: Euler was the first mathematician to introduce the symbol i (iota) for the square root of – 1 with property $i^2 = -1$. He also called this symbol as the imaginary unit.

- Iota (i) is neither 0, nor greater than 0, nor less than 0.
- The square root of a negative real number is called an imaginary unit.
- For any positive real number a, we have $\sqrt{-a} = \sqrt{-1 \times a} = \sqrt{-1}\sqrt{a} = i\sqrt{a}$
- $i\sqrt{-a} = -\sqrt{a}$.
- The property $\sqrt{a}\sqrt{b} = \sqrt{ab}$ is valid only if at least one of a and b is non-negative. If a and b are both negative then $\sqrt{a}\sqrt{b} = -\sqrt{ab}$.
- If $a < 0$ then $\sqrt{a} = \sqrt{|a|}\, i$.

(2) **Integral powers of iota (i):** Since $i=\sqrt{-1}$ hence we have $i^2=-1$, $i^3=-i$ and $i^4=1$. To find the value of $i^n\ (n>4)$, first divide n by 4. Let q be the quotient and r be the remainder.

i.e., $n=4q+r$ where $0\le r\le 3$

$i^n=i^{4q+r}=(i^4)^q.(i)^r=(1)^q.(i)^r=i^r$

In general we have the following results $i^{4n}=1,\ i^{4n+1}=i,\ i^{4n+2}=-1,\ i^{4n+3}=-i$, where n is any integer.

In other words, $i^n=(-1)^{n/2}$ if n is even integer and $i^n=(-1)^{n-1/2}i$ if n is odd integer.

The value of the negative integral powers of i are found as given below:

$$i^{-1}=\frac{1}{i}=\frac{i^3}{i^4}=i^3=-i, i^{-2}=\frac{1}{i^2}=\frac{1}{-1}=-1,\ i^{-3}=\frac{1}{i^3}=\frac{i}{i^4}=\frac{i}{1}=i,\ i^{-4}=\frac{1}{i^4}=\frac{1}{1}=1$$

Important Tips

- ☞ The sum of four consecutive powers of i is always zero i.e., $i^n+i^{n+1}+i^{n+2}+i^{n+3}=0,\ n\in I.$
- ☞ $i^n=1, i, -1, -i,$ where n is any integer.
- ☞ $(1+i)^2=2i,\ (1-i)^2=-2i$
- ☞ $\frac{1+i}{1-i}=i,\ \frac{1-i}{1+i}=-i,\ \frac{2i}{i-1}=1-i$