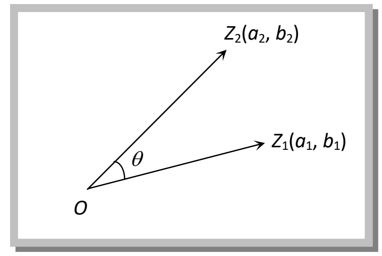
Dot and Cross Product.

Let $z_1 = a_1 + ib_1 \equiv (a_1, b_1)$ and $z_2 = a_2 + ib_2 \equiv (a_2, b_2)$ be two complex numbers.



If $\angle POQ = \theta$ then from coni method $\dfrac{z_2 - 0}{z_1 - 0} = \dfrac{|z_2|}{|z_1|} e^{i\theta}$

$\Rightarrow \dfrac{z_2 \bar{z}_1}{z_1 \bar{z}_1} = \dfrac{|z_2|}{|z_1|} e^{i\theta} \Rightarrow \dfrac{z_2 \bar{z}_1}{|z_1|^2} = \dfrac{|z_2|}{|z_1|} e^{i\theta} \Rightarrow z_2 \bar{z}_1 = |z_1||z_2| e^{i\theta}$

$\Rightarrow z_2 \bar{z}_1 = |z_1||z_2|(\cos\theta + i\sin\theta)$

$\Rightarrow \text{Re}(z_2 \bar{z}_1) = |z_1||z_2|\cos\theta$ .....(i) and $\text{Im}(z_2 \bar{z}_1) = |z_1||z_2|\sin\theta$ .....(ii)

The dot product z_1 and z_2 is defined by $z_1 o z_2 = |z_1||z_2|\cos\theta = \text{Re}(\bar{z}_1 z_2) = a_1 a_2 + b_1 b_2$ (From(i))

Cross product of z_1 and z_2 is defined by $z_1 \times z_2 = |z_1||z_2|\sin\theta = \text{Im}(\bar{z}_1 z_2) = a_1 b_2 - a_2 b_1$ (From(ii))

Hence, $z_1 o z_2 = a_1 a_2 + b_1 b_2 = \text{Re}(\bar{z}_1 z_2)$ and $z_1 \times z_2 = a_1 b_2 - a_2 b_1 = \text{Im}(\bar{z}_1 z_2)$

Important Tips

☞ If z_1 and z_2 are perpendicular then $z_1 o z_2 = 0$

☞ If z_1 and z_2 are parallel then $z_1 \times z_2 = 0$

☞ Projection of z_1 on $z_2 = (z_1 o z_2)/|z_2|$

☞ Projection of z_2 on $z_1 = (z_1 o z_2)/|z_1|$

☞ Area of triangle if two sides represented by z_1 and z_2 is $\frac{1}{2}|z_1 \times z_2|$

☞ Area of a parallelogram having sides z_1 and z_2 is $|z_1 \times z_2|$

☞ Area of parallelogram if diagonals represents by z_1 and z_2 is $\frac{1}{2}|z_1 \times z_2|$