Algebraic Operations with Complex Numbers.

Let two complex numbers $z_1 = a + ib$ and $z_2 = c + id$

Addition : $(a+ib)+(c+id)=(a+c)+i(b+d)$

Subtraction : $(a+ib)-(c+id)=(a-c)+i(b-d)$

Multiplication : $(a+ib)(c+id)=(ac-bd)+i(ad+bc)$

Division : $\dfrac{a+ib}{c+id}$ (when at least one of c and d is non-zero)

$$\frac{a+ib}{c+id}=\frac{(a+ib)}{(c+id)}.\frac{(c-id)}{(c-id)} \quad \text{(Rationalization)}$$

$$\frac{a+ib}{c+id}=\frac{(ac+bd)}{c^2+d^2}+\frac{i(bc-ad)}{c^2+d^2}.$$

Properties of algebraic operations with complex numbers: Let z_1, z_2 and z_3 are any complex numbers then their algebraic operation satisfy following operations:

(i) Addition of complex numbers satisfies the commutative and associative properties

i.e., $z_1 + z_2 = z_2 + z_1$ and $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$.

(ii) Multiplication of complex number satisfies the commutative and associative properties.

i.e., $z_1 z_2 = z_2 z_1$ and $(z_1 z_2) z_3 = z_1 (z_2 z_3)$.

(iii) Multiplication of complex numbers is distributive over addition

i.e., $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$ and $(z_2 + z_3) z_1 = z_2 z_1 + z_3 z_1$.

Note: $0 = 0 + 0i$ is the identity element for addition.

- $1 = 1 + 0\,i$ is the identity element for multiplication.
- The additive inverse of a complex number $z = a + ib$ is $-z$ (i.e. – a – ib).

For every non-zero complex number z, the multiplicative inverse of z is $\dfrac{1}{z}$.